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Tolerance Analysis in Mechanical Manufacturing Based on C-NPS

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Abstract: During the mechanical manufacturing process, parameters of part will fluctuate within the tolerance region. Assembly is composed of different components, regarding each component as an input; output will be a certain assembly function. Parts with tolerances assembled together inevitably create an uncertain circumstance in a mechanical assembly. Tolerance analysis is a very serviceable application for evaluating the accumulation of uncertainties caused by individual component tolerances. In this paper, a novel convex method and nonprobability set theory (C-NPS) are combined to model the variations of geometric features due to dimensional tolerances. This model is suitable for tolerance analysis of linear and nonlinear tolerances which are nonrandom parameters or unknown probability distribution in mechanical assemblies. The uncertainties in dimensions of features are mathematically illustrated using C-NPS model. Based on this method, a new nonprobability tolerance accumulation algorithm is created which accurately estimate the level of assembly uncertainty caused by mechanical manufacturing. In addition a novel optimization technique is introduced. The application of the proposed method is illustrated through presenting a Slider-Crank mechanism assembly problem, and its results are presented.

Keywords: mechanical manufacturing, tolerance analysis, component, assembly function, C-NPS

1. Introduction

Part with tolerance in Mechanical Manufacturing directly affects the finally performance and cost. As is known, assembly is composed of different components, regarding each component as an input; output will be a certain assembly function. Components assembled together inevitably create uncertainties in mechanical manufacturing. Tolerance analysis is an essential issue in the initial design tolerance; tolerance analysis methods are divided into two types which are worst case (W-C) method and statistical tolerance method, where most of these methods are based on the principle of probability and statistics. The W-C method is established based on interchangeable the part 100%. Each component dimension is assumed to be at its maximum, minimum or limit, resulting in the worst possible assembly limits [1]. Each component tolerance is a random variable, the results of worst case is inevitably conservative. Thus, with machining precision of parts improved, the manufacturing costs increase corresponding. Researchers usually put tolerance analysis of the uncertainty attributed to randomness. Khodaygan et al. [2] proposed a new method for estimation accumulative tolerances in a mechanical assembly. Marziale et al. [3] compared differences between Vector loop model and Matrix model. Beaucaire et at. [4] Adopted a method to evaluate a predicted quality level at the design stage based on statistical tolerance analysis, while Qureshi et al. [5] used same method for over-constraint mechanisms. Dantan et al. [6] simulated the influences of geometrical deviations on the geometrical behavior of the mechanism. Li et al. [7] constructed the Q-T functions for different categories of mechanisms based on small displacement torsor theory. The result of the W-C is overly pessimistic. In this case, the machining precision of parts should be improved, and the manufacturing cost increases at the same time. Statistical tolerance methods regard part variations of the assembly as randomness. Once the probability distribution is assumed, this can result in an unrealistic situation in many practical engineering. It will lose the significance of parameters design when probability distribution assumption is not satisfied.

This paper presents a convex-based tolerance representation scheme for modeling the uncertainty of tolerance in mechanical assemblies. This is a new method for tolerance analysis to deal with the contradictions above. In this paper, a novel method for modeling the uncertainty of tolerance mechanical assemblies is presented based on convex method and nonprobability set theory (C-NPS). In this model, uncertainties in dimensions and geometries of features due to tolerance are regard as nonrandom parameters and mathematically defined by convex set theory and a new nonprobability tolerance accumulation algorithm is created. This paper is organized as follows. In section II, the concepts involved as basis of the proposed method are described and the assembly function response is introduced. In section III, an example application is presented. Finally, section IV presents conclusions of the method.

2. Proposed method

2.1 Nonprobability set-theory

Modern scientific research shows that [8], the uncertainty is not necessarily random; the uncertainty is probably fuzzy or unknown but bounded. Moreover, information about random sample experiment on the probability distribution of size variation in tolerance range is often lacking or incomplete. Once those probability distribution assumption and tolerance of uncertain variables do not subject to the real distribution, then the rationality of the probability statistics analysis about tolerance lose significance.

Nonprobability set theory is different from probability theory, without knowing the probability distribution density of uncertain variables, merely required range of the uncertain variables. It has wide significance of set theory that if the parameters vary within a certain range is also regarded as a kind of uncertainty. Nonprobability set theory can accurately give bound to the final assembly response function. The scope of function responses compared to the probability distribution density is easier to determine.

2.2 Convex Method

The convex method was applied to model the uncertainties of variables. In some tolerances accumulation problems, the actual size (dimension) fluctuates between the upper and lower limits; the total range of the sizes in the assembly function can be seen as the input uncertainties. It might be suitable to aggregate all the uncertain-but-bounded tolerance variables into a group. In such circumstances, the Convex Method is competent for the description of these uncertainties.

In the space of parameters, any unknown parameter can be quantified to an ellipsoid according to Convex Method [9]. Thus assuming that the variation of unknown-but-bounded parameters vary around the nominal value can be described as

$$S = \{\xi^T \Omega \xi \leq \theta^2\} \tag{1}$$

Where, Ω is Positive Definite Matrix (PDM). θ is arithmetic number, $\theta \in [0,1]$.

In the parameter space of tolerance, with parameter uncertainty, assuming we only know their uncertainty range, and can be expressed in the following form of interval form as

$$X = [\underline{x}, \overline{x}] = \{x : \underline{x} \le x \le \overline{x}\} \qquad (2)$$

Where, $\overline{x}$ is the upper limits of the manufacturing tolerance, $\underline{x}$ is the lower limit of the manufacturing tolerance.

Supposing the uncertain-but-bounded variables of tolerances is gathered to a group denoted by:

$$f(X) = f(x_1, x_2, \cdots x_n) \qquad (3)$$

$f(X)$ is the assembly tolerances. The assembly tolerances are the allowances on the design requirements. The assembly function is the most essential equation for tolerance analysis and allocation that describes relations between the assembly and manufacturing tolerances.

All those geometric tolerances vary inside the Nonprobability Set can be solved using Convex Method from the geometrical significance of Convex Method. We will combine the Nonprobability Set theory and convex method together to build a novel model named C-NPS relay on the characters of these two conceptions. The details of C-NPS model will illustrate extensively in the section *C*.

2.3 Assembly function based on C-NPS

If the output result of assembly function is known, it could be convenient to conduct tolerance optimization in turn.

Provided $x^0 = (x_i^0)_n = (x_1^0, x_2^0, \ldots x_n^0)^T$ is median value of dimension with tolerance, the dimensional parameters can be represented as

$$x = x^0 + \xi \qquad (4)$$

Or

$$x_i = x_i^0 + \xi_i, \quad i = 1, 2, \cdots n \qquad (5)$$

Where,

$$\xi_i \le (\overline{x_i} - \underline{x_i})/2, \quad i = 1, 2, \cdots n \qquad (6)$$

The assembly function composed of n bounded dimensional parameters, then process this function to the Taylor expansion and reserved first items,

$$f(x) = f(x^0 + \delta) = f(x^0) + \sum_{i=1}^{n} \frac{\partial f(x^0)}{\partial x_i} \delta_i = f_0 + g^T \delta \tag{7}$$

Where,

$$f_0 = f(x^0) \tag{8}$$

$$g = \left(\frac{\partial f(x^0)}{\partial x_1}, \frac{\partial f(x^0)}{\partial x_2}, \cdots \frac{\partial f(x^0)}{\partial x_n}\right)^T \tag{9}$$

As the main issue of this work, we will not impose any distribution assumption about the input dimensional parameters $\{x_1, x_2, \dots, x_n\}$ and f.

The uncertain parameter δ varies inside boundary of Eq. 1; we can acquire the limits of the assembly function $f(x)$ through optimization techniques. So that the objective function is Eq. 7 and the constraint function is Eq.1.

$$\bar{f} = f_{\max} = \max_{\xi \in S(\xi, \theta)} \{f_0 + g^T \xi\} \tag{10}$$

$$\underline{f} = f_{\min} = \min_{\xi \in S(\xi, \theta)} \{f_0 + g^T \xi\} \tag{11}$$

Constructing the Lagrangian function of Eqs. 7, 10 and 11

$$L = f_0 + g^T \xi + \mu(\xi^T \Omega \xi - \theta^2) \tag{12}$$

Where μ is a Lagrangian multiplier? The necessary conditions for the extreme value is

$$\frac{\partial L}{\partial \xi} = g + 2\mu\Omega\xi = 0 \tag{13}$$

From the geometrical significance of Eq. 1, it can be converted to

$$S = \{\xi^T \Omega \xi \le \theta^2\} = \left\{\xi : \sum_{i=1}^{n} \frac{(\xi_i - \xi_i^0)^2}{e_i^2} \le \theta^2\right\} \tag{14}$$

The size parameter θ and the semi-axis vector $e = (e_1, e_2, \cdots, e_m)^T$ of the ellipsoid are obtained by means of the constraint condition of Eq. 2.

From equations above we can see that, the uncertain level of feature variations increases with the addition of dimensions and tolerances. It is necessary to lower the extent of input uncertainties for exacting assembly accumulation. Thus, the upper limit and lower limit of assembly function can be written as

$$\begin{cases} \overline{f} = f_{max} = f_0 + \Theta\sqrt{\sum_{i=1}^{n}\left(\frac{n}{2}\Delta x_i \frac{\partial f(x^0)}{\partial x_i}\right)^2} \\ \underline{f} = f_{min} = f_0 - \Theta\sqrt{\sum_{i=1}^{n}\left(\frac{n}{2}\Delta x_i \frac{\partial f(x^0)}{\partial x_i}\right)^2} \end{cases} \qquad (15)$$

Eq. 15 is the assembly function based on C-NPS. The equation gives out the limits of target assembly requirement.

As shown in Fig. 1, this strategy is employed to repeatedly change the uncertainty of inputs and whereby update the convex parameters until the satisfactory final assembly response is achieved.

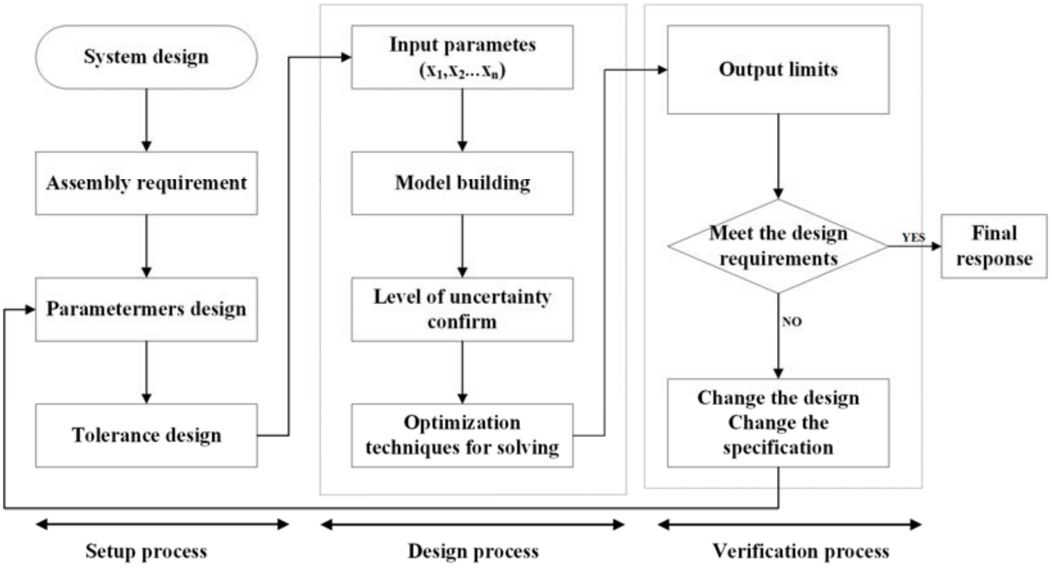

Fig. 1. Analysis process and feedback

3. Application: study of a sample assembly

3.1 Briefly introduction

To demonstrate the application of the proposed models, the example of Slider-Crank mechanism assembly is presented in detail below. In this example, the explicit design function can be expressed by a parametric relationship. For certification of computational results, we can use the results of direct dimensional analysis based on C-NPS model. The proposed method is applied to this Slider-Crank mechanism auto-assembly project. Consider the simple Slider-Crank mechanism assembly shown in Fig.2. This 3D assembly is made of four basic components: a crank, connecting rod, sliding block, and base frame. Rotary motion of the crank transform to the reciprocating linear motion of the slider. This assembly is widely applied in a variety of machine tools; crank rotates around the axis 360°. The coefficient of travel speed variation k is 1.5, when the crank rotates clockwise, the minimum transmission angle appear at the crank perpendicular to the horizontal moment. Using trigonometry, the dependent assembly function, angle Υ, can be expressed as explicit functions of X_1, X_2 and X_3 (see Fig.4). Υ is the minimum transmission angle. In order to guarantee the force transmission effect mechanism, must limit the minimum transmission angle of the mechanism in a certain range, that is,

$$\Upsilon_{min} \geq [\Upsilon] \tag{16}$$

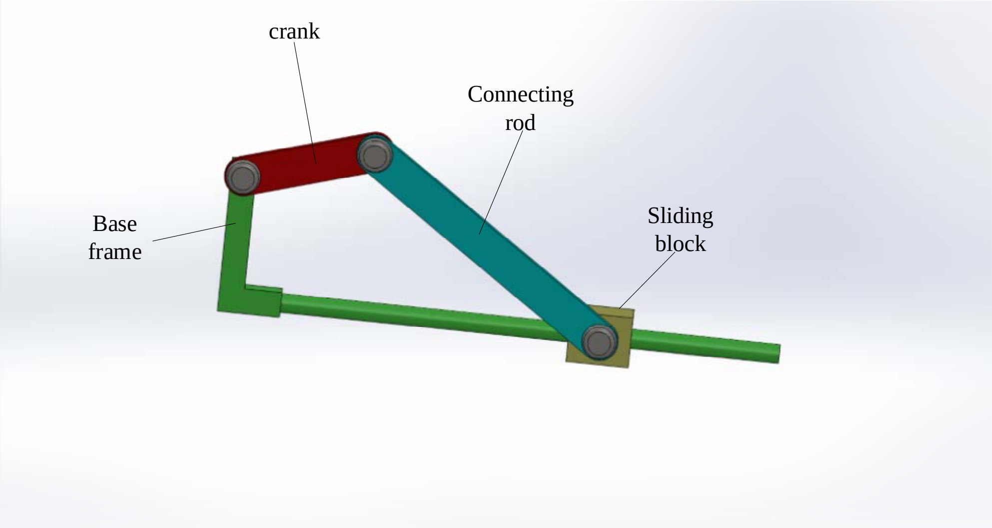

Fig. 2. Slider-Crank Mechanism

3.2 Method conduction

The expression for angle Υ may be analyzed statistically to estimate the resulting variation in Υ due to prescriptive tolerances for X_1 (length of crank), X_2 (eccentric distance), and X_3 (length of connecting rod), quantitatively. The nominal value of x_i and uncertain parameters of Δx_i, are given in Table 1.

$$\Upsilon = cos^{-1}\left(\frac{x_1+x_2}{x_3}\right) \tag{17}$$

$$\overline{\Upsilon} = \Upsilon_{max} = \Upsilon_0 + \Theta\sqrt{\sum_{i=1}^{3}\left(\frac{2}{3}\Delta x_i \frac{\partial f(x^*)}{\partial x_i}\right)^2} \tag{18}$$

$$\underline{\Upsilon} = \Upsilon_{min} = \Upsilon_0 - \Theta\sqrt{\sum_{i=1}^{3}\left(\frac{2}{3}\Delta x_i \frac{\partial f(x^*)}{\partial x_i}\right)^2} \tag{19}$$

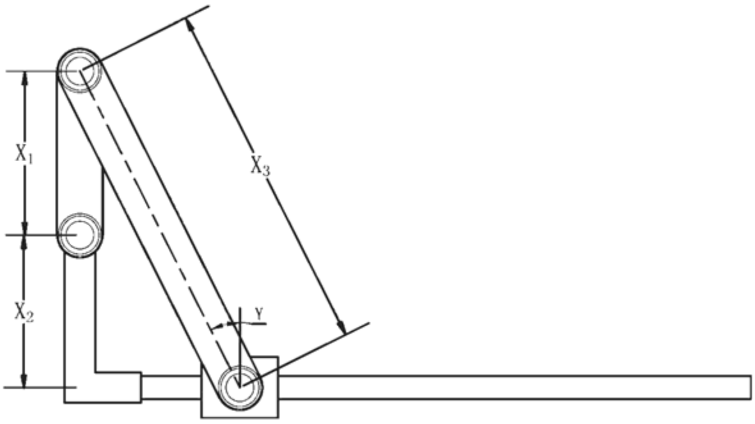

Fig. 3. γ is the Minimum Transmission Angle

TABLE I. SPECIFICATIONS OF MANUFACTURED VARIABLES OF SLIDER-CRANK ASSEMBLY

Variables	**Nominal**	**LL**	**UL**	x_i^0	Δx_i
x_1	21.5	-0.009	+0.012	21.5015	0.0105
x_2	20	+0.007	+0.02	20.0135	0.0065
x_3	46.5	-0.006	+0.01	46.502	0.008

TABLE I. shows the list of variables in Slider-Crank mechanism assembly, their normal, and their tolerance limits. For tolerance analysis based on C-NPS method, we can use Eq. 15 Substituting related numbers in Table 1 into Eqs. 18 and 19, the lower and upper tolerance limits of the assembly variable (Y) are acquired. The results are shown in Fig. 4.

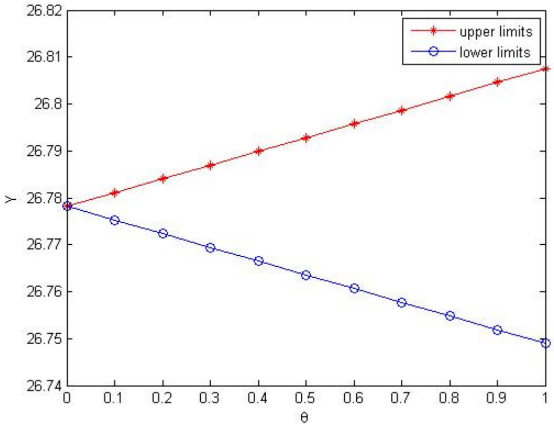

Fig. 4. Results of Y based on C-NPS method

It is accessible to confirm the range of uncertain variables distribution than the density of those variables. Nonprobability set theory can give out the assembly function response range or robustness margin easily. Final response range of assembly function is prone to determine than the probability distribution. It should be emphasized that the set theoretical convex method does not exclude the problem of uncertainty in tolerance analysis using probability processing; on the contrary, the method we proposed can be seen as a supplementary method of probabilistic model to study the uncertainty problems.

4. Conclusion

This is a new method for tolerance analysis in mechanical manufacturing and the research of us makes up for the deficiency in addition to statistical analysis. The two main concepts, nonprobability set theory and convex method, are introduced to expound uncertain variables in assembly tolerances of unknown probability distribution. The main advantage of nonprobability set theory is different from probability theory, without knowing the probability distribution density of uncertain variables, which can be given out the final responses of assembly function quickly and accurately. It is accessible to confirm the range of uncertain variables than the distribution density of those variables. The proposed method considers all the influential dimensional variations in analysis. This method can be applied both in linear and nonlinear assembly functions with ease.

Acknowledgments

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