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Matlab-based Modeling and Simulation Experiment of Segway System

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Abstract: Based on Segway's movement diagram and Lagrangian equation, the author builds and linearizes a systematic mathematical model and dynamic structural diagram. And its dynamic situation is reflected through Matlab simulation. Moreover, with the pendulum angle as its inner loop, Segway displacement designs a PD control system for its outer loop as well as Matlab simulation. Due to its feedback control, the PD Control System acts as a powerful guarantee for Segway's balance in motion.

Keywords: Segway; Systemic modeling; PD control; Matlab simulation

1. Introduction

Nowadays, industries, aerospace industries and human entertainment have posed new problems and challenges to control technologies. Under such circumstances, control theories have made great progress and got wide applications. Segway is an unprecedented brand-new two-wheeled transport device powered by electrics. It is driven by electric motors and keeps its balance through coordination of various mechanical devices. By human body center's changes, Segway can make such movements as starting, speeding, decelerating, stopping, etc. There are varied self-control systems. To analyze it through simulation, it's necessary to model the system's movement routines with mathematical formulae firstly and then apply Matlab to design simulation so that we can get real simulation results.

2. Systematic Modeling

Segway is composed of a vehicle body and two wheels. Its body and man's center of gravity are inverted on the above of its wheel axes. Segway can move while keeping its balance. It can also walk upright. The wheels are not only affected by electric motor output torque, ground support and varied resistance, but also by its body's force through electric motor axes.

To make it easy to analyze its systematic principles, Segway can be simplified as the model shown in Fig.1, without considering air resistance, frictions and other elements existential in real environments [1] [2]. The paper focuses on the electric scooter's linear movement situation.

As shown in Fig.1, let's suppose that: Mass of pendulum rod and man is m_1, wheel mass is m_2, length of pendulum rod and man is $2l$, oscillation angle of pendulum rod and man is θ_1, wheel angle is θ_2, speed of pendulum rod and man in the X direction is V_{1X} and that in the Y direction V_{1Y}, wheel speed in the X direction is V_{2X} and that in the Y direction V_{2Y}, inertia of pendulum rod and man is J_1 and wheel inertia J_2. (In kinematics, in case of $\dot{\theta} = \frac{d\theta}{dt}$, given each derivative of the function of time t, we will add a dot above the symbol of the function.)

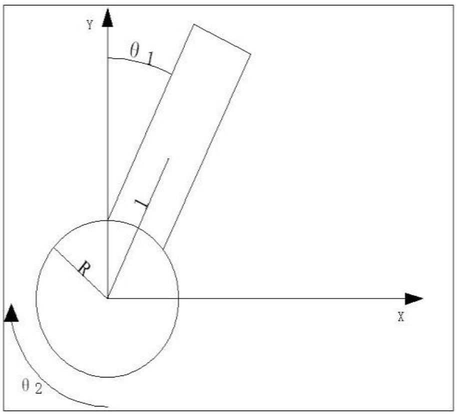

Fig.1. Schematic Segway

Lagrange equation is: $\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_i}) - \frac{\partial L}{\partial q_i} = Q_i$. (1)

From the kinematic perspective, we build the system model with Lagrange equation. There are two degrees of freedom in the two-wheeled electric device in linear motion. That is, Segway winding axis' back and forth swinging and translational motion in horizontal plane. The paper selects oscillation angle of pendulum rod and man θ_1 and wheel angle θ_2 as generalized coordinates, among which, q_i is generalized coordinate, $q_1 = \theta_1$, $q_2 = \theta_2$; while $L = T - V$ is Lagrange function, the value of difference of the system's kinetic energy and potential energy; and Q_i is non-potential generalized force.

As the system's all generalized coordinates are oscillation angles, non-potential generalized forces are torsional moment supposed as M_1, M_2.

To model by Lagrangian equation, we should: firstly, solve speeds of each part; and secondly, solve the system' kinetic energy, gravitational potential energy and generalized forces.

From Fig.1, we know that Speed of pendulum rod and man is $V_{1X} = V_{2X} + l\dot{\theta}_1 \cos\theta_1$, $V_{1Y} = -l\dot{\theta}_1 \sin\theta_1$, wheel speed is $V_{2X} = R\dot{\theta}_2$, $V_{2Y} = 0$ .Kinetic energy of pendulum rod and man is

$$T_1 = \frac{1}{2}m_1(V_{1X}^2 + V_{1Y}^2) + \frac{1}{2}J_1\dot{\theta}_1^2 = \frac{1}{2}m_1R^2\dot{\theta}_2^2 + \frac{1}{2}m_1l^2\dot{\theta}_1^2 + m_1Rl\dot{\theta}_1\dot{\theta}_2\cos\theta_1 + \frac{1}{2}J_1\dot{\theta}_1^2 .$$

Kinetic energy of wheels is

$$T_2 = \frac{1}{2}m_2V_2^2 + \frac{1}{2}J_2\dot{\theta}_2^2 = \frac{1}{2}m_2R^2\dot{\theta}_2^2 + \frac{1}{2}J_2\dot{\theta}_2^2 .$$

Total kinetic energy of the system is

$$T = T_1 + T_2 = \frac{1}{2}(m_1l^2 + J_1)\dot{\theta}_1^2 + \frac{1}{2}(m_1R^2 + m_2R^2 + J_2)\dot{\theta}_2^2 + m_1Rl\dot{\theta}_1\dot{\theta}_2\cos\theta_1 . \quad (2)$$

Suppose $K_1 = m_1l^2 + J_1$, $K_2 = m_1R^2 + m_2R^2 + J_2$, put it in Formula (2) and get

$$T = T_1 + T_2 = \frac{1}{2}K_1\dot{\theta}_1^2 + \frac{1}{2}K_2\dot{\theta}_2^2 + m_1Rl\dot{\theta}_1\dot{\theta}_2\cos\theta_1 . \quad (3)$$

Based on analysis of the system, potential energy of pendulum rod and man is

$$V = m_1gl\cos\theta_1 \quad (4)$$

Put $L = T - V$ into Formula (3) and (4) and get

$$L = \frac{1}{2}K_1\dot{\theta}_1^2 + \frac{1}{2}K_2\dot{\theta}_2^2 + m_1Rl\dot{\theta}_1\dot{\theta}_2\cos\theta_1 - m_1gl\cos\theta_1 \quad (5)$$

While solving non-potential generalized forces, we can suppose there is some little movement δ in the system. And then in the little movement change, the system work is $\delta W = 0\cdot\delta\theta_1 + M\cdot\delta\theta_2$.

Due to $\delta W = M_1\cdot\delta\theta_1 + M_2\cdot\theta_2$, since $M_1 = 0$, $M_2 = M$ .Therefore, the Lagrange equation is

$$\begin{cases} \frac{d}{dt}(\frac{\partial L}{\partial\dot{\theta}_1}) - \frac{\partial L}{\partial\theta_1} = 0 \\ \frac{d}{dt}(\frac{\partial L}{\partial\dot{\theta}_2}) - \frac{\partial L}{\partial\theta_2} = M \end{cases} . \quad (6)$$

Based on Formula (5), we get

$$\frac{\partial L}{\partial\dot{\theta}_1} = K_1\dot{\theta}_1 + m_1Rl\dot{\theta}_2\cos\theta_1 ,$$

$$\frac{d}{dt}(\frac{\partial L}{\partial\dot{\theta}_1}) = K_1\ddot{\theta}_1 + m_1Rl\ddot{\theta}_2\cos\theta_1 - m_1RL\dot{\theta}_1\dot{\theta}_2\sin\theta_1 , \quad (7)$$

$$\frac{\partial L}{\partial\theta_1} = -m_1Rl\dot{\theta}_1\dot{\theta}_2\sin\theta_1 + m_1gl\sin\theta_1 . \quad (8)$$

According to Formula (7) and Formula (8), we get

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}_1})-\frac{\partial L}{\partial \theta_1}=K_1\ddot{\theta}_1+m_1Rl\ddot{\theta}_2\cos\theta_1-m_1gl\sin\theta_1 . \quad (9)$$

Based on Formula (5) again, we get

$$\frac{\partial L}{\partial \dot{\theta}_2}=K_2\dot{\theta}_2+m_1Rl\dot{\theta}_1\cos\theta_1 ,$$

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}_2})=K_2\ddot{\theta}_2+m_1Rl\ddot{\theta}_1\cos\theta_1-m_1Rl\dot{\theta}_1^2\sin\theta_1 , \quad (10)$$

$$\frac{\partial L}{\partial \theta_2}=0 . \quad (11)$$

According to Formula (10) and (11), we get

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}_2})-\frac{\partial L}{\partial \theta_2}=K_2\ddot{\theta}_2+m_1Rl\ddot{\theta}_1\cos\theta_1-m_1Rl\dot{\theta}_1^2\sin\theta_1 . \quad (12)$$

Wheel torsional moments are supplied by electric motor which bears a relationship with control voltage:

$$M=K_mU . \quad (13)$$

Here, U is added control voltage and K_m is moment coefficient of electric motor. Combine Formula (6), (9), (12) and (13), we get

$$\begin{cases} K_1\ddot{\theta}_1+m_1Rl\ddot{\theta}_2\cos\theta_1-m_1gl\sin\theta_1=0 \\ K_2\ddot{\theta}_2+m_1Rl\ddot{\theta}_1\cos\theta_1-m_1Rl\dot{\theta}_1^2\sin\theta_1=K_mU \end{cases} \quad (14)$$

3. Simplified Model

Given that human body stands nearly upright on Segway, value range of θ_1 is $-10^\circ<\theta_1<10^\circ$. Therefore, we can approximate $\dot{\theta}_1^2\approx 0$, $\sin\theta_1\approx\theta_1$, $\cos\theta_1\approx 1$.

And then Formula (14) can be simplified as

$$\begin{cases} K_1\ddot{\theta}_1+m_1Rl\ddot{\theta}_2-m_1gl\theta_1=0 \\ K_2\ddot{\theta}_2+m_1Rl\ddot{\theta}_1=K_mU \end{cases} \quad (15)$$

Since we cannot access Segway's detailed data, we substitute it with parameters of two-wheeled balancing robots of the company Oupeng, as shown in Table 1[3].

Table 1. Parameters of Two-wheeled Balancing Robots of the Company Oupeng

Symbol	Name	Unit	Numerical Value
m_1	Pendulum Rod Mass	kg	10
m_2	Wheel Mass	kg	1.6
l	Length from Center of Mass to Axis	m	0.1
R	Wheel Radius	m	0.075
J_1	Moment of Inertia	$kg\cdot m^2$	0.1
J_2	Wheel Moment of Inertia	$kg\cdot m^2$	0.0036
K_m	Moment Coefficient	vs/rad	0.0364
g	Gravitational acceleration	N/kg	9.8

Based on $K_1=m_1l^2+J_1$ $K_2=m_1R^2+m_2R^2+J_2$, we get $K_1=0.2$ $K_2=0.06885$.

Put parameters into Formula (15), we get

$$\begin{cases} 0.2\ddot{\theta}_1 + 0.075\ddot{\theta}_2 - 9.8\theta_1 = 0 \\ 0.06885\ddot{\theta}_2 + 0.075\ddot{\theta}_1 = 0.0364U \end{cases}.$$

We use Laplace transform to the above Formula and get the system's transfer function as follows:

$$\begin{cases} G_1(s) = \dfrac{\theta_1(s)}{U(s)} = \dfrac{-0.335175}{s^2 - 82.839779} \\ G_2(s) = \dfrac{\theta_2(s)}{\theta_1(s)} = \dfrac{-0.2s^2 + 9.8}{0.75s^2} \end{cases}.$$

4. Modeling Verification

A system transfer structural diagram is built by transfer function, as is shown in Fig.2.

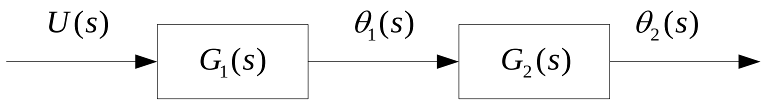

Fig.2. System Transfer Structural Diagram

Add a voltage 4V suddenly to Segway and last 0.5s. According to past experience, the pendulum rod will fall down and the device will move forward. We apply Simulink in Matlab to simulate and get Fig.3 and Fig.4.

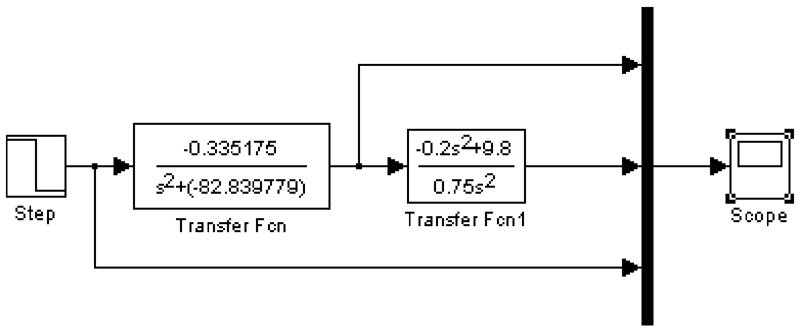

Fig.3. Simulation Diagram of Verification Mode

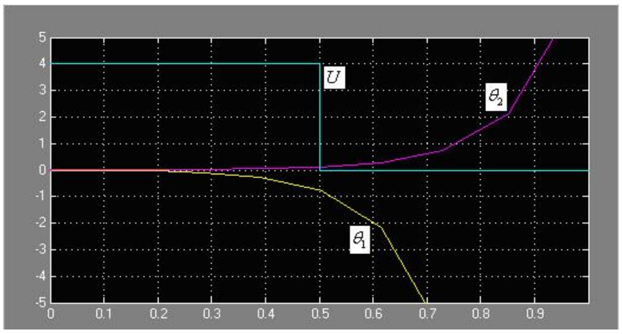

Fig.4. this is a verification simulation results diagram: blue lines represents Voltage U, yellow lines θ_1 and red lines θ_2.

Image displaying belongs to one of our common movement conditions. Therefore, the mathematic modeling is successful.

5. Controller Design

System Simulation Diagram with Forward Channel adding proportional element $D(s) = -100$ and inputting θ_1 =0.1rad, as is shown in Fig.5 [4].

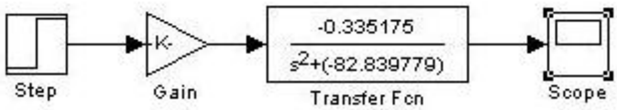

Fig.5. System Simulation Diagram

Obviously, from Fig.6, we find the output angle θ_1 is continuously increasing, so the subject is a non-linear system with natural instability. Therefore, we decide to apply feedback control, as is shown in Fig.7.

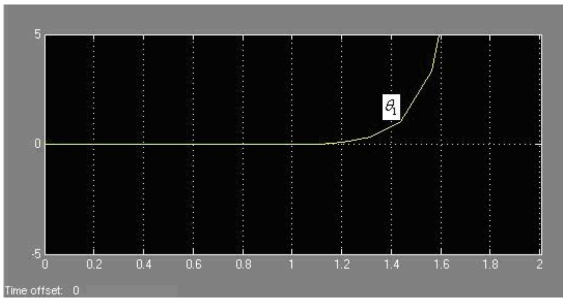

Fig.6. System Simulation Results

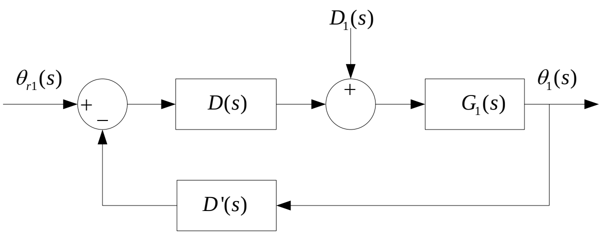

Fig.7. Feedback Control Frame Diagram

Feedback controller is $D'(s)$. The paper selects PD controller[5]. Using PD-structured feedback controller can stabilize the previously unstable system. And we add a proportional element $D(s) = K$ in forward channel to strengthen the control ability of the disturbance variable $D_1(s)$.

As for the confirmation of controller parameters, we temporarily define the proportional element as $D(s) = -100$, and then we can deduct the transfer function:

$$W=\frac{KG_1(s)}{1+KG_1(s)D'(s)}=\frac{-100\times\frac{-0.335175}{s^2-82.839779}}{1+(-100)\times\frac{-0.335175}{s^2-82.839779}(K_P+K_d s)}=\frac{33.5175}{s^2+33.5175K_d s+33.5175K_p-82.839779}.$$

Obviously, this is a typical second-order system. As the system asks for no special index demands, we apply typical parameter determination methods. That is, the methods can ensure that feedback system is characteristic of rapid following performance traits (make the damping proportion $\xi=0.7$ and closed-loop gain $K=1$).

We apply these conditions to confirm feedback controller parameters K_p and K_d, and get

$$\begin{cases}33.5175K_p-82.839779=33.5175\\33.5175K_d=2\times0.7\times\sqrt{33.5175}\end{cases}.$$

From the above formula, we get $\begin{cases}K_p=3.4715\\K_d=0.24182\end{cases}$.

After getting K_p and K_d, we can then simulate the control system by using Simulink, as is shown in Fig. 8.

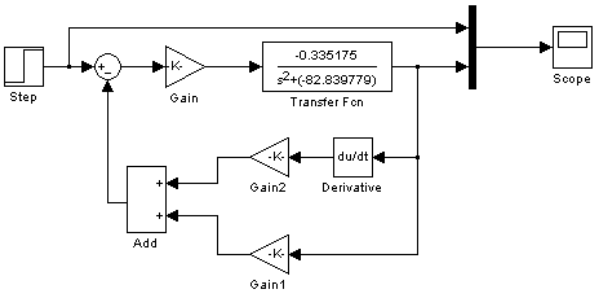

Fig.8. Feedback Control System Using Simulink Simulation

Input θ_1=0.1rad in the first second and we get simulation results, as is shown in Fig.9. It is easy to notice that the yellow line overlaps the red line from the secondary second and is close to non-oscillation, therefore, it realizes stability. This proves that the system design is very successful.

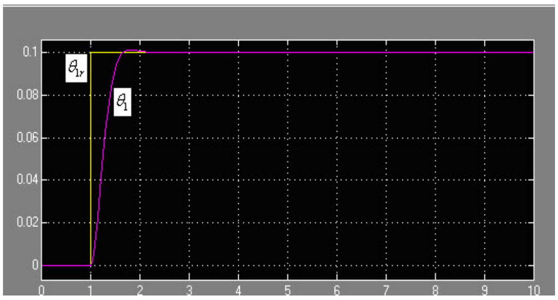

Fig.9. Yellow lines represent input angle θ_{1r} while red lines output angle θ_1.

6. System Simulation and Movement Situation Observation

After feedback controller is designed, the whole control system can then be established, whose simulation structural diagram is shown in Fig.10.

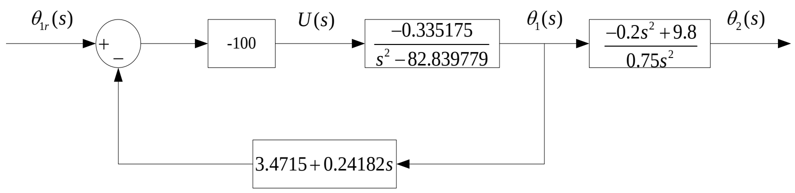

Fig.10. Simulation Structural Diagram

Suppose the initial angle is 0.1rad and lasts for 1 second, and then we turn it into -0.1rad to observe the situation of θ_1 and θ_2.
Simulate the system by using Simulink, as is shown in Fig.11.

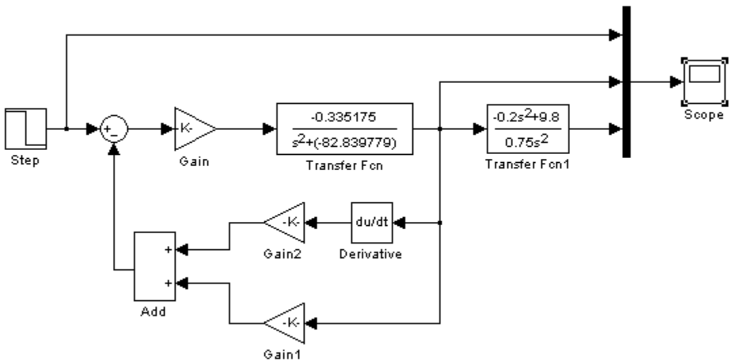

Fig.11. Simulate the System by using Simulink

From Fig.12, it is observed that when the red line is positive, the device is accelerating; when the red line is negative, it is decelerating. This can well explain Segway's working principles. That is, man leans forward to accelerate, balances himself to move at constant speed, and leans backward to decelerate.

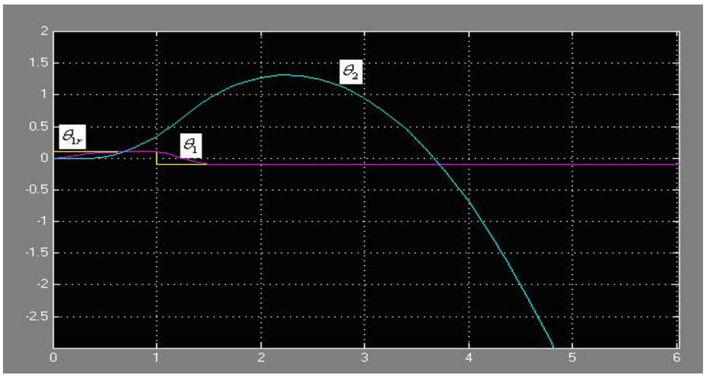

Fig.12. Yellow Lines represents Input θ_{1r} , while Red Line Output θ_1 and Blue Line θ_2 .

7. Summary

Based on Segway movement principles, the paper conducted mathematic modeling to the system, applied relevant theory of automatic control system to design control and simulated the model by using Simulink in Matlab. According to simulation results, the author concludes that the vehicle's constant speed, acceleration and deceleration can be achieved by human body's standing upright, leaning forward and backward. The experiment taps into knowledge from different disciplines to finish simulation coordination, which boasts of the following advantages: we can observe control results of the controlled subject through Matlab and make varied algorithm adjustments without building real objects, thus greatly shorten the research and development period and reduce cost of many automatic machine inventions. Through the experiment, we prove that there is an emerging development tendency that takes advantages of one research field in combination with other different disciplines.

Acknowledgements

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