Steady Harmonic Signal Analysis of Power System Based on Prony Spectral Line Estimation and Neural Network Algorithm

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Abstract: A steady-state harmonic signal analysis method for power system based on Prony spectral line estimation algorithm and neural network is proposed to improve the accuracy of harmonic and harmonic steady-state signal analysis. The Prony spectral line estimation algorithm has the excellent feature of directly obtaining the signal frequency. Firstly, Prony spectrum estimation algorithm is used to estimate the frequency components of the signal, and the number of neural networks and initial parameters are determined. Second, the neural network is used to estimate the phase and amplitude of the fundamental and harmonic waves. MATLAB simulation results show that the algorithm achieves high accuracy and fast convergence.

Keywords: Power System, Prony, ADALINE, Steady-State Harmonic, Signal Analysis.

1. Introduction

In today's society, with the rapid development and widespread application of various power electronic devices, the effects of harmonics and inter-harmonics in power systems are widespread and increasingly serious. Various operational faults caused by harmonics and inter-harmonics in power systems and Safety accidents also occur frequently, and the seriousness of harmonics and inter-harmonics hazards in the power system gradually attracts people's attention and becomes an important problem that the power system has to face. The harmonics of the power system are characterized by randomness, nonlinearity, distribution, flight stability, and influencing factors. The study of harmonics in the power system has always been a research hotspot and difficulty in the power system [1].

Current methods for power system signal analysis mainly include Fast Fourier Transform (FFT), Wavelet Transform, Regression (AR) Spectral Estimation, Prony Algorithm, and Neural Network Method. Using FFT requires longer sample data but
lower frequency resolution. Wavelet transform can reflect the time-varying characteristics of the signal and also require longer sampling data [2]. In addition, it is difficult to choose a wavelet basis. The AR spectrum estimation method has higher frequency resolution, but the position of the peak is vulnerable to the initial phase of the signal, and the phenomenon of “split line” appears. The Prony spectral estimation algorithm model has the characteristics of accurately describing the power signal, and directly acquires the frequency of the signal when short-time data is needed, and accumulates rounding errors when calculating the amplitude and initial angle of each level [3]. Neural networks have achieved good results in harmonic measurements. However, when harmonics are present, the number and value of frequency components are needed to determine the network structure and initial training values. In [4], ADALINE neural network and windowed interpolation FFT were proposed for harmonic analysis. This method improves the measurement accuracy but does not overcome the shortcomings of FFT.

In this paper, we use Prony spectral estimation in conjunction with neural network algorithms to detect inter-harmonics. Firstly, the Prony spectral line estimation algorithm is used to detect the frequency of the signal, which is used to obtain the number of neurons and the frequency parameters of the neural network. Next, consider each frequency as input to a neural network to estimate the phase and amplitude of all harmonic signals. The paper is organized as follows: Section 2 introduces the Prony spectral line estimation and ADALINE adaptive neural network algorithm. Section 3 discusses some simulation results, while Section 4 provides some conclusions and future research.

2. Prony Spectral Line Estimation Algorithm

The mathematical model used by the Prony spectral line estimation algorithm is a set of $p$ real, non-attenuating sine functions whose discrete time functions [5]:

$$\hat{x}(n) = \sum_{i=1}^{p} \cos(2\pi f_i n + \theta_i)$$

(1)

Where $\hat{x}(n)$ is an approximation of $x(n)$.

The key to the Prony spectral line estimation algorithm is to realize that the fit of equation (1) is a homogeneous solution of a linear difference equation with constant coefficients [6, 7]. The derivation process is as follows:

Harmonic processes can be described by difference equations. Consider the case of a single sine wave [8]. From trigonometric identity transformation

$$\cos(2\pi f n + \theta) + \cos(2\pi f (n-2) + \theta) = 2\cos(2\pi f) \cos(2\pi f (n-1) + \theta)$$

(2)
If \( x(n) = \sum_{i=0}^{p} \cos(2\pi f n + \theta_i) \) is substituted into the above equation, the second-order difference equation can be obtained.

\[
x(n) - 2\cos(2\pi f) x(n-1) + x(n-2) = 0
\]

(3)

Z-transform on the above

\[
[1 - 2\cos(2\pi f) z^{-1} + z^{-2}]X(z) = 0
\]

(4)

Then, get the characteristic polynomial

\[
1 - 2\cos(2\pi f) z^{-1} + z^{-2} = 0
\]

(5)

It has a pair of conjugated plural roots

\[
z = \cos(2\pi f) \pm j\sin(2\pi f) = e^{\pm j2\pi f}
\]

(6)

Note that the moduli of conjugate roots are 1, that is, \(|z_1| = |z_2| = 1\) which determine the frequency of the sine wave:

\[
f_i = \arctan(\text{Im}(z_i)/\text{Re}(z_i))/2\pi
\]

(7)

Usually, just take the positive frequency. Obviously, if \( p \) real sine wave signals have no repetition frequency, then the \( p \) frequencies should be determined by the root of the characteristic polynomial

\[
\prod_{i=1}^{p} (z - z_i)(z - z_i^*) = \sum_{i=0}^{2p} a_i z^{2p-i} = 0
\]

(8)

Or

\[
1 + a_1 z^{-1} + \cdots + a_{2p-1} z^{-(2p-1)} + z^{-2p} = 0
\]

(9)

It is easy to know that all of these roots are equal to one. Since all roots are in the form of conjugate pairs, the coefficients of the characteristic polynomial (9) have symmetry

\[
a_i = a_{2p-i} \quad i = 0, 1, \ldots p
\]

(10)

The difference equation corresponding to equation (9) is

\[
x(n) + \sum_{i=1}^{2p} a_i x(n-i) = 0
\]

(11)

The error between the actual measured data \( x(n) \) and its approximate value \( \hat{x}(n) \) is defined as \( e(n) \):

\[
x(n) = -\sum_{i=1}^{2p} a_i \hat{x}(n-i) + e(n)
\]

(12)

\[
= -\sum_{i=1}^{2p} a_i \hat{x}(n-i) + \sum_{i=0}^{2p} a_i e(n-i) \quad n = 0, 1, \ldots N - 1
\]

Now, the criterion for least-squares estimation of parameter \( a_i, \ldots, a_{2p} \) is to minimize \( \sum_{n=0}^{N-1} |e(n)|^2 \).

\[
e(n) = \sum_{i=0}^{n} a_i [x(n+i) + x(n-i)]
\]

(13)
Where $a_i = a_{i-1}$, because $a_i = a_{2p-i}$

$$
\sum_{n=0}^{N-1} |e(n)|^2 = \sum_{n=0}^{N-1} \sum_{i=0}^{p} |\tilde{a}[x(n+i) + x(n-i)]|^2
$$

By minimizing the sum of squared errors in the above equation

$$
\sum_{i=0}^{p} \tilde{a} r(i, j) = 0
$$

$$
r(i, j) = \sum_{n=p}^{N-1} [x(n+j) + x(n-j)][x^*(n+i) + x^*(n-i)]
$$

In this way, the coefficient value can be estimated, the root of the difference equation can be found, and the frequency value of the signal can be estimated.

### 3. ADALINE Neural Network Analysis Model

The structure of the ADALINE neural network is shown in Figure 1:

![ADALINE neural network schematic](image)

The input vector is $X = [\sin(2\pi f_1 n), \cos(2\pi f_1 n), \ldots, \sin(2\pi f_p n), \cos(2\pi f_p n)]^T$, and the weight vector is $W = [a_1, b_1, \ldots, a_p, b_p]$. $y(n)$ is the actual output value of the neural network, $\hat{x}(n)$ is the target output value of the neural network, and the basic expression of the actual output value of the neural network is:

$$
e(n) = \hat{x}(n) - y(n)
$$

In order to make the actual output value and the target output value infinitely close, the algorithm uses the least mean square algorithm (LMS) to adjust the weights.

### 4. Simulation Analysis

In order to verify the proposed Prony-based and neural network-based algorithm, the simulation signal is as follows:

$$
x(n) = \sum_{m=1}^{q} A_m \cos(2\pi f_m n\Delta t + \theta_m) + \xi(n)
$$
In the above equation, $\xi(n)$ is Gaussian white noise, the signal to noise ratio is 50dB, and the sampling frequency is 3000Hz. Other parameters such as frequency, amplitude and phase of frequency components are shown in Table 1.

4.1 Signal frequency estimation using Prony algorithm
The Prony algorithm has an order of $p=10$, which is twice the number of frequency components. According to the Prony algorithm, the singular value decomposition (SVD) algorithm is first used to solve the AR model, and then the least squares method is used to estimate the frequency of the differential signal [9]. Table 2 lists the Prony algorithm's estimate of the frequency and the estimated error rate.

<table>
<thead>
<tr>
<th>Component</th>
<th>$f_0$ = 50</th>
<th>$f_1$ = 95</th>
<th>$f_2$ = 200</th>
<th>$f_3$ = 465</th>
<th>$f_4$ = 650</th>
</tr>
</thead>
<tbody>
<tr>
<td>F(Hz)</td>
<td>49.25</td>
<td>93.57</td>
<td>199.52</td>
<td>464.88</td>
<td>650.2</td>
</tr>
<tr>
<td>error</td>
<td>1.5%</td>
<td>1.5%</td>
<td>0.24%</td>
<td>0.047%</td>
<td>0.031%</td>
</tr>
</tbody>
</table>

4.2 Signal Parameter Estimation Based on ADALINE Neural Network Algorithm
This section uses the ADALINE neural network algorithm to estimate the amplitude and phase based on the frequency obtained by the Prony algorithm. The number of neurons is 10, and the learning rate of the neural network is $\eta = 0.002$. After many iterations, the convergence curve is as shown in the figure:

Fig 2. ADALINE neural network convergence curve
5. Conclusion

For the characteristics of harmonic signals between power systems, using Prony spectral line estimation algorithm to detect the frequency of the signal, it can exert its extremely high spectral resolution characteristics. Using ADALINE neural network to calculate the amplitude and phase can avoid the calculation caused by Prony algorithm. Accumulation of errors. The simulation results of MATLAB show that the algorithm has higher accuracy and faster convergence speed. However, how to eliminate the effect of Gaussian white noise is a problem that needs further study.

References


