

Periodic Gait and Chaos in the Biped Passive Walking Model

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Abstract: Based on the theory of passive dynamic walking, analyze the stable period gait of the Compass model, established the mathematical model of the walking process. The influence of different parameter changes on the gait of the model was analyzed by the combination of theoretical and numerical calculation. By calculating the complex dynamic behavior of the system, it provides a theoretical basis for the actual construction of the model and the control of walking gait.

Keywords: Dynamic passive walking; poincare map; chaos.

1. Introduction

The study of humanoid robot mainly focuses on human-like walking, and the stability of the robot motion and energy dissipation problem is very important. Currently, researchers used more active dynamic walking style, which has a high control precision, but the cost is much expensive, and it has a heave quality. The quasi-passive dynamic walking with swing and hitting the ground switching process, without control among swing, is natural and in lower energy consumption. But the passive dynamic walking is mainly used for the analysis of the complex gait and the theoretical control algorithm at present, but with very little consideration of the influence of parameters on the system. The Compass-like model, which proposed by Goswami, is the most representative of the human leg structure in the passive walking model.

2. Mathematic modeling

We consider a very simple model of a compass-like, planar, biped robot as shown in Fig.1. φ is the angle of the gentle slope. The compass-like model, which composed of two completely identical legs: same quality and geometry parameters of swing leg and

stance leg, and a frictionless hip to connecting the two legs. The two legs are modeled as rigid bars without knees and feet [1]. The compass-gait biped robot consists of nonlinear differential equations for the swing stage and algebraic equations for the impact stage [2]. We make $\boldsymbol{\theta} = [\theta_{ns}, \theta_s]^T$ the vector of generalized coordinates of the compass-gait model [3].

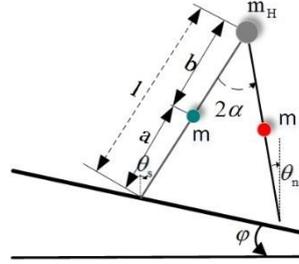


Fig. 1 Compass Like Passive Dynamic Walking Model

The swing motion which similar to that of a frictionless double pendulum, can be described by the equations have the following form:

$$M(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + N(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + G(\boldsymbol{\theta}) = 0 \quad (1)$$

$$M(\boldsymbol{\theta}) = \begin{bmatrix} mb^2 & -mlb \cos(\theta_s - \theta_{ns}) \\ -mlb \cos(\theta_s - \theta_{ns}) & (m_H + m)l^2 + ma^2 \end{bmatrix} \quad (2)$$

$$N(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \begin{bmatrix} mlb\dot{\theta}_s^2 \sin(\theta_s - \theta_{ns}) \\ -mlb\dot{\theta}_{ns}^2 \sin(\theta_s - \theta_{ns}) \end{bmatrix} \quad (3)$$

$$G(\boldsymbol{\theta}) = g \begin{bmatrix} mb \sin(\theta_{ns}) \\ -(m_H l + m(a + l)) \sin(\theta_s) \end{bmatrix} \quad (4)$$

At the moment of collision, the Compass model swaps the roles of the two legs, the swing leg is converted into the next support leg, and the strut leg is converted into the swing leg, and the conversion process is Collision process of model dynamic system, which has $\alpha = \frac{1}{2}(\theta_s - \theta_{ns})$.

$$Q_m(\alpha) = \begin{bmatrix} -mab & -mab + (m_H l^2 + 2mal) \cos(2\alpha) \\ 0 & -mab \end{bmatrix}$$

$$Q_p(\alpha) = \begin{bmatrix} mb(b - l \cos(2\alpha)) & ml(l - b \cos(2\alpha)) + ma^2 + m_H l^2 \\ mb^2 & -mbl \cos(2\alpha) \end{bmatrix}$$

3. Numerical Simulation

The basic idea of Poincaré mapping is to construct a Poincaré section intersecting the periodic trajectory to obtain the intersection point on the section. The stability of the equilibrium point on this Poincaré section is equivalent to the periodic trajectory

stability[4]. The stability of the system periodic trajectory can be determined by the method of judging the stability of the balance point. For the Compass-like model, system dynamics can be represented by state variable values at any one time in a cycle. In the Compass-like model, during the walking process, the roles of the swinging leg and the supporting leg are mutually converted, and the switching moment is selected as the Poincaré cross section, and the switching surface equation of the system is:

$$P = \left\{ (\theta_{ns}, \theta_s, \dot{\theta}_{ns}, \dot{\theta}_s) \mid \theta_{ns} + \theta_s = -2\varphi, \sin(\theta_{ns}) - \sin(\theta_s) > 0 \right\}$$

For the Poincaré section, the Poincaré mapping corresponding to the model's oscillating equation can be defined as follows: Randomly select a point on the Poincaré section to make a point. Under an initial condition, the point that the point returns first after a certain period of time is. The equation of the system continuous time is calculated first, until the system solution hits the Poincaré section at the moment, it will be brought into the collision phase equation, and it will be used as the next initial value to continue the iterative operation.

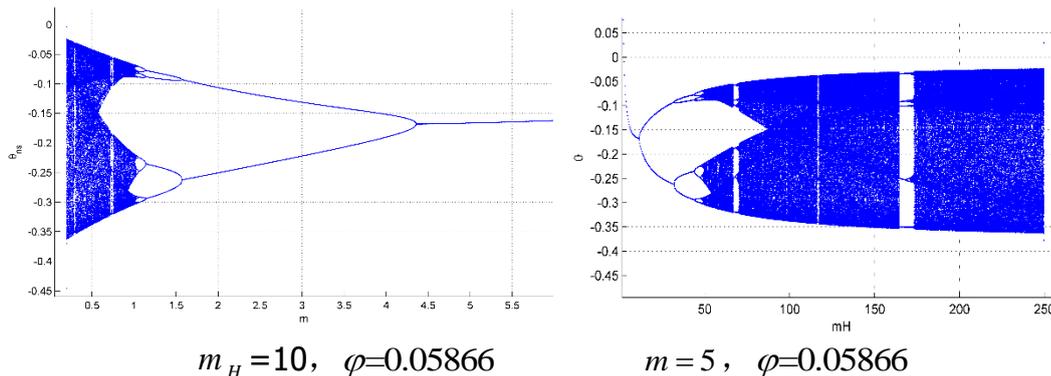


Fig. 2 Compass Like Passive Model shown different bifurcation

When $m_H = 10$ fix the hip mass and change the weight of the leg. At that time, the system showed a two-cycle bifurcation phenomenon when $m = 4.356$, and as m decreased, the system eventually reached chaos. As can be seen from the bifurcation diagram, the angle does not change significantly as it decreases. At the time $m_H = 11.1$, the system generates two cycles.

With the increase, the phenomenon of tillingering eventually reaches chaos. It can be clearly seen that the angle decreases as the two-cycle gait occurs. It indicates that the hip mass is inversely proportional to the step size. Goswami found that, as the tilt angle increases, Compass-like model will be from a gait cycle by bifurcations into the two-fold or greater cycle period.

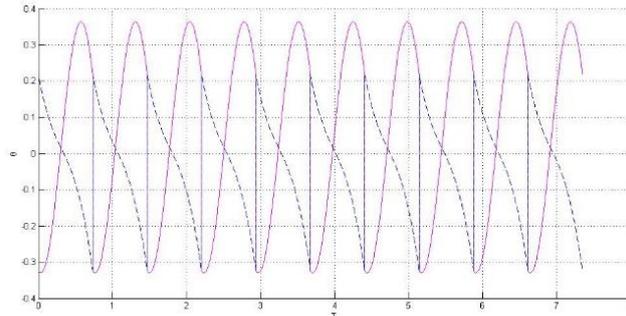


Fig. 3 The angle change diagram of the swing leg and the support leg

The parameters of the group are as follows $a=1, b=1, m=5, m_H = 10, \varphi=0.05234$ and the initial value $\theta_{ns} = -0.32339, \theta_s = 0.21867, \dot{\theta}_{ns} = -0.37718, \dot{\theta}_s = -1.0918$ is a stable gait of one cycle at this time, and Figure 3 is the angle change diagram of the swing leg and the support leg.

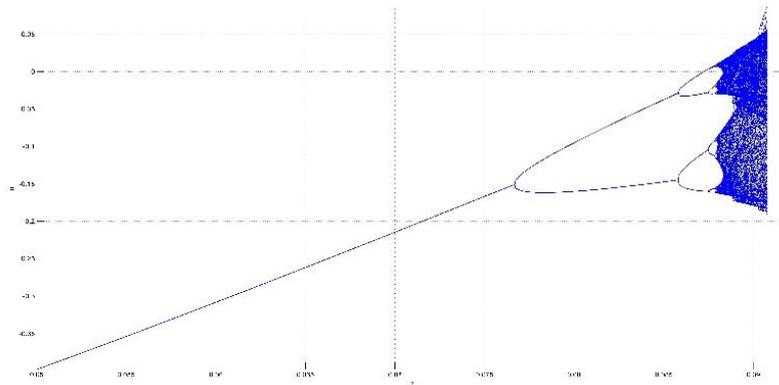


Fig. 4 The phenomenon of bifurcation

As shown in the figure 4, the simulation diagram of the change of φ during the change is gradually increased. The gait of Compass passive walking is chaotic by the period of one cycle and the phenomenon of bifurcation. It can be seen that as the slope angle becomes larger, a one-cycle gait appears bifurcation. The gait cycle times after bifurcation into a two-cycle gait at $\varphi=0.07668$ then when $\varphi=0.08579$, two destabilized gait cycle, entering the fourth period gait; With the change, if it continues to increase nearby $\varphi = 0.086$ rad. The above simulation shows that in the process of increasing the slope, the model will gradually enter the chaotic state, which will eventually lead to the robot not being able to walk stably or even fall.

Through numerical simulation, the effects of the height of the leg mass, the mass ratio of the hip and the leg mass and the change of the slope angle on the passive gait were found. Comparative experiments showed that with the center of mass ratio, mass ratio, the slope angle of three sets of parameters changes, the system stabilizing Compass Point Fixed fold bifurcation phenomena occur, ultimately chaos.

4. Conclusion

In this paper, the motion process of the compass model is deeply studied. The changes of the stable walking gait of the robot under the parameter change are discussed. The existence conditions of the stable walking cycle of the robot are analyzed. The stable periodic gait of the model is described along with its different parameters. The change situation, and pointed out that a stable gait in a cycle can achieve chaotic state by double-cycle bifurcation. The passive walking model adopted in this paper is an idealized model. There are still problems such as collision and friction in actual walking. How to effectively limit the model parameters, how to optimize the system initial value to make the system have a larger attraction domain. And the production of a more practical system is the next step in the research.

References

- [1] Gritli H, Khraief N, Belghith S. Chaos control in passive walking dynamics of a compass-gait model[J]. *Communications in Nonlinear Science & Numerical Simulation*, 2013, 18(8):2048-2065.
- [2] Moon J S, Spong M W. Bifurcations and chaos in passive walking of a compass-gait biped with asymmetries[C]// *IEEE International Conference on Robotics & Automation*. 2010.
- [3] Gritli H, Khraief N, Belghith S. Period-three route to chaos induced by a cyclic-fold bifurcation in passive dynamic walking of a compass-gait biped robot[J]. *Communications in Nonlinear Science & Numerical Simulation*, 2012, 17(11):4356-4372.
- [4] Segre B. Computation of the Lyapunov exponents in the compass-gait model under OGY control via a hybrid Poincaré map[J]. *Chaos Solitons & Fractals*, 2015, 81(s 6–7):172-183.