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Research on Multi-task Principal-agent Model Based on Reliability Factors of Production Management System

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Abstract: It is assumed that the operational reliability of the production management system of an enterprise implicates the mutual knowledge of the principal-agent parties. Here, the influence of this internal and external uncertainty factors on the enterprise operation under this condition are taken into account. An incentive mechanism model for business operators accepting multiple tasks is established using an incomplete information dynamic game. Through analysis of the optimal solution of the model, under the circumstance in which the incentive costs of multitasking efforts are independent of each other, the optimal reward for each task of the operator, the optimal reward for the operator's immeasurable effort to complete the task, and the influence of relevant parameters on the optimal solution are discussed, followed by the conclusions.

Keywords: Multitask, incentive mechanism, reliability, dynamic game, principal-agent.

1. Introduction

Issues implicating the incentives of business operators in a single-task principal-agent relationship have been studied from many aspects, such as agency mode, the optimal combination of long-term and short-term rewards, a combination of immaterial and material incentives and the incentive effect.

However, in real-life situations, the principal-agent relationship is often multitasked instead of single-tasked. Moreover, since the working competence required from enterprise agents may be irrelevant, it is unrealistic to require them to be equally efficient and comprehensive in all tasks. Therefore, it is necessary to adopt a new approach to study the incentives of enterprise agents in a multi-task principal-agent relationship.

Concerning the incentives of enterprise agents in a multi-task principal-agent relationship, Holmstrom has studied the incentive contracts and work arrangements of business operators in such a multi-task principal-agent relationship, indicating that conclusions drawn from the simple principal-agent model are inapplicable to a multitask principal-agent relationship. They have provided a new analytical tool for academics to further study the incentive problem under the more complex principal-agent relationship. Baker (1988,1992) [1-2] has studied the impact of enterprise performance evaluation on the multitask principal-agent incentive mechanism. Tian Ying and Pu Yongjian (2006) [3]have studied the optimal design of the incentive mechanism in a multitask principal-agent relationship between business operators and owners by using the constant elasticity of substitution (CES) to represent the income of the principal.

Yuan Jiangtian and Zhang Wei (2006) [4]have established a multitask principal-agent model for SOE managers under the administrative appointment system. Based on an analysis of the incentive problems of managers who have accepted three tasks, the optimal incentive contract for managers of state-owned enterprises under this condition was obtained. Gong (2017) [5] and Carroll (2017) [6] further relaxed the conditions and reached a conclusion with wider applicability. Liu Rui and Yu Qinhua (2015) [7] and Ohad (2017) [8] considered the correlation of incentives and concluded that with an increase in mutual substitution among tasks, incentive forces must be further enhanced.

By analyzing the limitations of the above models, it can be seen that above-mentioned models reflect the inevitable limitations, namely, it ignores enterprise system reliability and other internal factors. Through a study of the multi-task incentive model about business operators, the author tries to analyze the incentive issues on the premise of considering enterprise system reliability.

2. Mechanical Analysis

2.1 Establishment of model and optimal solution

The certainty equivalence income of business operators is

$$CE = Es(x) - \frac{1}{2}\rho\beta^{\mathrm{T}}\Pi\beta - c(a_1,\ldots,a_n)$$

$$= \alpha + \beta^{\mathrm{T}}(m_1a_1,\ldots,m_na_n)^T - \frac{1}{2}\rho\beta^{\mathrm{T}}\Pi\beta - c(a_1,\ldots,a_n) \tag{1}$$

The expected profit of the entrusting party is

$$B(a_1,\ldots,a_n) - Es(x) = B(a_1,\ldots,a_n) - \alpha - \beta^{\mathrm{T}}(m_1a_1,\ldots,m_na_n)^T \tag{2}$$

The fixed remuneration for business operators α refers to the income that they obtain regardless of how they work, which therefore does not affect the coefficient vector β during multitasking and the effort vector of the business operator $(a_1,...,a_n)$ towards a specific number of tasks (n tasks). Under a given β and $(a_1,...,a_n)$, fixed remuneration α may be determined by the reservation utility of the business operator. Both α and $\beta^{\mathrm{T}} \cdot u(a_1,...,a_n)$ are part of the wealth created by the operator for the enterprise. They only affect the distribution of the wealth created by the business operator between the principal and the business operator. Π is the covariance matrix of vector β.

In summary, in a multi-task principal-agent relationship, the incentive model about business operators considering the system reliability factors of the enterprise is constructed as follows:

$$\underset{\beta}{Max} TCE = B(a_1,...,a_n) - \frac{1}{2}\rho\beta^{\mathrm{T}}\Pi\beta - c(a_1,...,a_n)$$

$$s.t \ (a_1,...,a_n) \in \arg\max\left\{\alpha + \beta^{\mathrm{T}} \cdot u(a_1,...,a_n) - \frac{1}{2}\rho\beta^{\mathrm{T}}\Pi\beta - c(a_1,...,a_n)\right\} \quad (3)$$

Assuming that the effort the operator makes to complete Task Number i is strictly greater than zero, $i = 1,...,n$, the constraint conditions of the business operator in Equation (3) can be simplified as follows:

$$\beta_i = \frac{c_i(a_1,...,a_n)}{m_i} \ i = 1,...,n \quad (4)$$

In the above formula, $c_i(a_1,...,a_n) = \dfrac{\partial c(a_1,...,a_n)}{\partial a_i}$, $c_{ij} = \dfrac{\partial c_i(a_1,...,a_n)}{\partial a_j}$

The partial derivative with respect to a_i in Equation (4) is calculated and the results are as follows:

$$\frac{\partial \beta}{\partial a} = \left(\frac{c_{ij}}{m_i}\right), \ \frac{\partial a}{\partial \beta} = \left(\frac{c_{ij}}{m_i}\right)^{-1} \quad (5)$$

$$\frac{\partial \beta}{\partial a} = \begin{pmatrix} \frac{\partial \beta_1}{\partial a_1},...,\frac{\partial \beta_1}{\partial a_n} \\ ... \\ \frac{\partial \beta_n}{\partial a_1},...,\frac{\partial \beta_n}{\partial a_n} \end{pmatrix}, \ \left(c_{ij}\right) = \begin{pmatrix} c_{11},...,c_{1n} \\ ... \\ c_{n1},...,c_{nn} \end{pmatrix} \quad (6)$$

Use the above conditions and set the first partial derivative of the operator's total certainty equivalence income with respect β as 0. The results are set forth below:

$$\frac{\partial B}{\partial a} \cdot \frac{\partial a}{\partial \beta} - \rho\Pi\beta - \frac{\partial c}{\partial a} \cdot \frac{\partial a}{\partial \beta} = 0$$

$$\frac{\partial B}{\partial a} - \rho \frac{\partial \beta}{\partial a} \Pi \beta - \frac{\partial c}{\partial a} = 0$$

It can be seen from (4) and (5) that

$$B' - \rho M^{-1}(c_{ij}) \Pi \beta - M \beta = 0$$

$$B' = \left(\rho M^{-1}(c_{ij}) \Pi + M\right) \beta$$

$$\beta = \left(\rho M^{-1}(c_{ij}) \Pi + M\right)^{-1} B' \qquad (7)$$

In the above formula,

$$M = \begin{pmatrix} m_1, 0, \ldots, 0 \\ \ddots \\ 0, \ldots, 0, m_n \end{pmatrix}$$

$$M^{-1} = \begin{pmatrix} \frac{1}{m_1}, 0, \ldots, 0 \\ \ddots \\ 0, \ldots, 0, \frac{1}{m_n} \end{pmatrix}$$

$B' = (B_1, \ldots, B_n)^{\mathrm{T}}$, $B_i = \frac{\partial B}{\partial a_i}$, $i = 1, \ldots, n$. B_i is the marginal revenue of a business operator's efforts for Task Number i.

2. 2 Incentive issues affecting business operators when their effort costs are independent of each other

2.2.1 Solution of the model

To facilitate the analysis of the above mentioned issue, further assumptions are made as follows:

Hypothesis: When a business operator completes a certain number of tasks (n tasks), the effort costs are independent of each other, and the random factor vectors that affect the performance of the business operator are independent identically distributed. That is, $c_{ij} = 0$, $i \neq j$ and the covariance among ε_i is 0, say $r_{ij} = 0$, $i \neq j$.

Although this hypothesis has particularity, it is established in the face of provisions and facts such as the "one-vote veto system for safety liability accidents" and "a few insiders making a manager selection".

In the case of this hypothesis, Π is the diagonal matrix, and

$$\Pi = \begin{pmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_n^2 \end{pmatrix}$$

At this point, from $\beta_i = \frac{c_i(a_1,...,a_n)}{m_i}$, $i = 1,...,n$, it can be seen that

$$\frac{\partial \beta}{\partial a} = \begin{pmatrix} \frac{\partial \beta_1}{\partial a_1} & 0 & \cdots & 0 \\ 0 & \frac{\partial \beta_2}{\partial a_2} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \frac{\partial \beta_n}{\partial a_n} \end{pmatrix} = \begin{pmatrix} \frac{c_{11}}{m_1} & 0 & \cdots & 0 \\ 0 & \frac{c_{22}}{m_2} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \frac{c_{nn}}{m_n} \end{pmatrix}$$

Put $\Pi = \begin{pmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_n^2 \end{pmatrix}$ into Equation (7), and the results are as shown below:

$$\beta = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_n \end{pmatrix} = \left[\begin{pmatrix} m_1 & 0 & \cdots & 0 \\ 0 & m_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & m_n \end{pmatrix} + \rho \begin{pmatrix} \frac{1}{m_1} & 0 & \cdots & 0 \\ 0 & \frac{1}{m_2} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{m_n} \end{pmatrix} \begin{pmatrix} c_{11} & 0 & \cdots & 0 \\ 0 & c_{22} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & c_{nn} \end{pmatrix} \begin{pmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_n^2 \end{pmatrix} \right]^{-1} \begin{pmatrix} B_1 \\ \vdots \\ B_n \end{pmatrix}$$

$$= \begin{pmatrix} \frac{m_1}{m_1^2 + \rho c_{11}\sigma_1^2} & 0 & \cdots & 0 \\ 0 & \frac{m_2}{m_2^2 + \rho c_{22}\sigma_2^2} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \frac{m_n}{m_n^2 + \rho c_{nn}\sigma_n^2} \end{pmatrix} \cdot \begin{pmatrix} B_1 \\ \vdots \\ B_n \end{pmatrix}$$

$$= \begin{pmatrix} \frac{m_1 B_1}{m_1^2 + \rho c_{11}\sigma_1^2} \\ \vdots \\ \frac{m_n B_n}{m_n^2 + \rho c_{nn}\sigma_n^2} \end{pmatrix}$$

In summary, it can be obtained that

$$\beta_i^* = \frac{m_i B_i}{m_i^2 + \rho c_{ii}\sigma_i^2} \quad i = 1,...,n \qquad (8)$$

In the formula above, β_i^* decreases with the increase of ρ, c_{ii} and σ_i^2.

Based on the above analysis, our conclusions are set forth below.

Proposition: When the proposition satisfies the assumptions and conditions for the model described in this paper, under the condition of incentive compatibility, the optimal merit pay obtained by the business operator is independent of each task in terms of the mutually independent information among the performance of each effort and under the condition in which the effort costs of each task are independent of each other. The optimal merit pay obtained by the business operator is the decreasing function of absolute risk aversion rate ρ ,the rate of change of the marginal effort costs c_{ii} and the observable variance of natural state σ_i^2.

From the above demonstration, it can be seen that under the conditions of meeting the above assumptions and propositions, the optimal merit pay of the business operator for the multitasks it has accepted is independent of each task. The reward received by the business operator for each task is exclusively related to the effort for the task.

2.2.2 Comparative static analysis of relevant parameters

The negative correlation (not linear) between each merit pay β_i^* and absolute risk aversion ρ means that the more averse the business operator is to the risk, the less incentive the enterprise owner should give the business operator when formulating incentive schemes. Therefore, business operators with different risk attitudes should be given different optimal incentive schemes for the multitasks they accept. Of course, in practice, blindly meeting the individual needs of business operators for different performance and different tasks will result in a lack of unity in incentive policies. The issue of how to coordinate different risk attitudes among business operators toward multitasks is a difficult operator incentive problem. Moreover, compared to older people who are ready to retire, young business operators may have a higher risk preference, considering that they will be given more opportunities in the future. Therefore, incentives for young operators while completing tasks should be reduced. In contrast, for older operators, incentives to complete tasks should be enhanced.

As seen from the relationship between β_i^* and the rate of change of the marginal incentive cost c_{ii}, if the change rate of unit incentive cost of the task faced by the business operator is high, the entrusting party shall reduce the incentive on the completion of the task. Otherwise, the incentive on the completion of the task should be increased. This theory reveals the incentive policy principle to state-owned business operators.

The negative correlation between the optimal merit pay β_i^* and the variance of observable variables σ_i^2 corresponding to the number of tasks(n tasks) reflects that when the effort made by a business operator for a specific task is strongly related to business performance, that is, when the variance of the observable variables of the task is very low, the entrusting person can stimulate the operator to work hard to maximize the operator's benefits by enhancing the incentive for the task. The reason behind this is that if the variance of the observable variables of a certain task is high, it indicates that the effort made by the business operator on the task is not necessarily related to performance. The operator's good performance may be due to chance and opportunity, which cannot be used as the basis to truly reflect effort level and work competence.

Considering the relationship between the operational reliability index of the production management system of an enterprise m_i and the optimal merit pay of the operator β_i^*, it can be seen that

$$\frac{\partial \beta_i}{\partial m_i} = \frac{B_i\left(\rho c_{ii}\sigma_i^2 - m_i^2\right)}{\left(m_i^2 + \rho c_{ii}\sigma_i^2\right)^2}$$

The conclusion of the above equation is

$$\begin{cases} \dfrac{\partial \beta_i}{\partial m_i} > 0, 0 < m_i < \sqrt{\rho c_{ii}\sigma_i^2} \\ \dfrac{\partial \beta_i}{\partial m_i} \le 0, m_i \ge \sqrt{\rho c_{ii}\sigma_i^2} \end{cases}$$

It can be seen that when $0 < m_i \le \sqrt{\rho c_{ii}\sigma_i^2}$, $\frac{\partial \beta_i}{\partial m_i} > 0$,the optimal reward of the enterprise operator increases with the increase of the operational reliability index of enterprise system related to this task. However, when $m_i > \sqrt{\rho c_{ii}\sigma_i^2}$, $\frac{\partial \beta_i}{\partial m_i} < 0$, the operator's blind pursuit for high reliability in enterprise system operation will create more production and management costs to the enterprise. At this point, the optimal reward of the business operator decreases with the increase in the reliability index mi related to the task.

3. Conclusion

Through the analysis of the influence of enterprise system reliability index on the operator's optimal incentive reward, it is concluded that in the early stage of operation, the operation reliability index of the enterprise system can indirectly reflect the management effect of the business operator. If the index rises, the operator's reward has a positive impact on the performance of the enterprise. The entrusting party should therefore consider increasing the incentive to the business operator. However, with the continuous increase in the enterprise system reliability index mi, the operating cost of the enterprise will inevitably increase. When the added value of the operating cost exceeds the added value of the operating performance of the enterprise, the entrusting party will reduce the incentive remuneration of the operator under the condition that the total operating cost of the enterprise remains unchanged from the perspective of self-interest maximization.

This study further indicates that in production, operation and management, excellent business operators should have a global awareness and reasonably arrange various tasks under the premise of coordinating the operation of the enterprise system. Only in this way can the enterprise vigorously maintain its vitality in the face of fierce market competition.

On incentive issues affecting business operators, there are other research directions in the future when the effort costs are interrelated of each other.

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