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The Batch Calculation of Parameters of Super-stable Period Sequences in Five Letters of Symbolic Dynamics Based on The High Precision NTL Libary

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Abstract: The word-lifting technique is applied to calculate the parameters of quadruply super-stable periodic kneading sequences (QSSKS), if a QSSKS is admissible, the corresponding system of nonlinear equations must be solvable. A golden initial value is determined by the geometrical property of the five letters system, it works well with low-period cases, however, when the period is long, a few QSSKSs can not be calculated correctly which are called admissible bad QSSKSs. In the paper, we research these bad QSSKSs, 86% of them are affected by the error accumulations, when we apply the high-precision algorithm from a library for doing number theory (NTL) into the word-lifting technique, the numeric solution will reach the genuine one. The rest is still related to the more accurate initial values.

Keywords: Initial Value, High-Precision Algorithm, Fixed Point, Ntl Library.

1. Introduction

Symbolic dynamics is an important tool to explore many fields [1]. The QSSKSs may construct the kneading space of quad-modal maps, the parameter calculation of the QSSKSs is an important way to explore the metric universalities on the bifurcations to chaos [2], the word-lifting technique acquired fast development to provide a powerful support [3]. An admissible QSSKS determines a solvable system of nonlinear equations. By newton iteration method, we have found the selection of initial values and forward-iteration method have played an important role in the numeric solution [4].

The so-called golden initial point can be determined by the geometrical property of quad-modal maps. By the initial point $P_0 = (-0.72591, -11829, 0.16225, 0.72324)$, about 86% system of nonlinear equations from the admissible QSSKSs may converge into the fixed point, in another word, they meet the symbolic convergence condition by the forward-iteration method and numeric convergence condition by the newton iteration method. Those parameters can not be attained by the golden initial point, the corresponding QSSKSs are called "bad QSSKSs". In this paper, the bad QSSKSs are thus explored by numeric experiments. One of the reasons for no solution come from the error accumulations, in fact, in spite of being the contraction mapping of the inverse branches in the word-lifting technique, however, the error accumulations can not be omitted yet for some long-period QSSKSs. The solution for the error accumulations is to apply the high-precision algorithm from the famous NTL library with the word-lifting technique, the result is many of the bad QSSKSs have attained the fix points during the parameter computation. Finally, we have found only 5% bad QSSKSs still have a strong relation with the initial points, the so-called golden initial point is not universal, the golden property inside the unimodal maps could come from the simplicity of the mappings. The touchy of the initial points in the numeric solution is still worthy of exploring in the word-lifting technique.

2. A QSSKS and Its System of Nonlinear Equations

2.1 The Symbolic Dynamics of Five-Letters

The mapping come from 1D mapping (2.1):

$$f(x,p) = k\int (x-c_1)(x-c_2)(x-c_3)(x-c_4)dx + \lambda \tag{2.1}$$

We apply the boundary conditions $f(-1,p) = -1$ and $f(1,p) = 1$, here $p = (c_1, c_2, c_3, c_4, k, \lambda)$, the two parameters k and λ may be eliminated, so (2.1) may lead to an iterative mapping (2.2):

$$x_{n+1} = f(x_n, c_1, c_2, c_3, c_4) \tag{2.2}$$

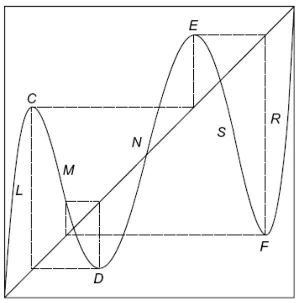

Fig. 1 The schematic of the orbit iterative

Assume c_1, c_2, c_3 and c_4 are horizontal coordinates of four critical points C, D, E and F respectively. For an initial point x_0, we get a series of orbit points $x_1, x_2, \cdots, x_n, \cdots$, these points may be coarse-graining symbolized by the following rules:

$$x_i \text{ is denoted as } \begin{cases} L, & \text{if } -1 < x_i < c_1 \\ C, & \text{if } x_i = c_1 \\ M, & \text{if } c_1 < x_i < c_2 \\ D, & \text{if } x_i = c_2 \\ N, & \text{if } c_2 < x_i < c_3 \\ E, & \text{if } x_i = c_3 \\ S, & \text{if } c_3 < x_i < c_4 \\ F, & \text{if } x_i = c_4 \\ R, & \text{if } c_4 < x_i < 1 \end{cases} \tag{2.3}$$

Here， a numeric orbit is translated into a symbolic sequence by (2.3), besides four critical point C,D,E and F, five monotonous limbs are denoted as five symbols L,M,N,S and R, so (2.2) may be called symbolic dynamics of five letters after the symbolization (2.3). The periodic sequences which pass through all the four critical points are call super-stable kneading sequences and simply noted as QSSKSs. Mackay and Tress [5] called them as the skeletons of the kneading space. It is important to explore the parameters of the QSSKSs.

Admissible sets of n-period QSSKSs may be produced by the permutation of symbols and admissible conditions (2.4),

$$\begin{aligned} &\bar{L}(W) < \bar{C}(W),\ \bar{D}(W) < \bar{M}(W) < \bar{C}(W),\ \bar{D}(W) < \bar{N}(W) < \bar{E}(W), \\ &\bar{F}(W) < \bar{S}(W) < \bar{E}(W),\ \ \bar{F}(W) < \bar{R}(W). \end{aligned} \tag{2.4}$$

W is a sequence under detection, if $X \in \{L, C, M, D, N, E, S, F, R\}$, then $\bar{X}(W)$ stands for the subsequent sequence for every letter X in the W. The order rules of the symbolic sequences may refer to the book [6], here is omitted for concision. In table 1, the number of admissible QSSKSs period from 4 to 11 are presented, all 217617 QSSKSs are calculated for their parameters in this paper.

Table 1 The number of admissible QSSKSs period from 4 to 11

period	Four types of QSSKs				Total
	FEDC	FDEC	EFDC	DFEC	
4	0	0	1	0	1
5	2	0	8	0	10
6	15	4	42	0	61
7	88	33	200	2	323
8	467	209	907	24	1607
9	2349	1162	4026	194	7731
10	11414	6068	17754	1306	36542
11	54457	30497	78392	7996	171342

2.2 System of Nonlinear Equations Determined by a QSSKS

Here, QSSKS $RMFLSNESDC$ is admissible, according to the word-lifting technique and periodicity, (2.5) may be obtained,

$$\begin{cases} f(c_1,c_1,c_2,c_3,c_4) = f_R^{-1}\circ f_M^{-1}(c_4,c_1,c_2,c_3,c_4), \\ f(c_4,c_1,c_2,c_3,c_4) = f_L^{-1}\circ f_S^{-1}\circ f_N^{-1}(c_3,c_1,c_2,c_3,c_4), \\ f(c_3,c_1,c_2,c_3,c_4) = f_S^{-1}(c_2,c_1,c_2,c_3,c_4), \\ f(c_2,c_1,c_2,c_3,c_4) = c_1. \end{cases} \tag{2.5}$$

(2.5) is written as (2.6) equivalently,

$$\begin{cases} \varphi_1(c_1,c_2,c_3,c_4) = f(c_1,c_1,c_2,c_3,c_4) - f_R^{-1}\circ f_M^{-1}(c_4,c_1,c_2,c_3,c_4) = 0, \\ \varphi_2(c_1,c_2,c_3,c_4) = f(c_4,c_1,c_2,c_3,c_4) - f_L^{-1}\circ f_S^{-1}\circ f_N^{-1}(c_3,c_1,c_2,c_3,c_4) = 0, \\ \varphi_3(c_1,c_2,c_3,c_4) = f(c_3,c_1,c_2,c_3,c_4) - f_S^{-1}(c_2,c_1,c_2,c_3,c_4) = 0, \\ \varphi_4(c_1,c_2,c_3,c_4) = f(c_2,c_1,c_2,c_3,c_4) - c_1 = 0. \end{cases} \tag{2.6}$$

So, for a proper initial point $P_0 = (c_1^0, c_2^0, c_3^0, c_4^0)$, the system of nonlinear equations (2.6) determined by a QSSKS $RMFLSNESDC$ can be solved by Newton method for a numeric solution, for this example, the solution is (-0.798143666206774, -0.289928229572866, 0.301066871677701, 0.793394680358814).

3. The Golden Initial Points and Application of The NTL Library

3.1 The Method to Get the Golden Initial Points

From the mapping (2.1) and its geometrical property, the following conditions (3.1) should be satisfied:

$$\begin{cases} f(-1,p) = -1,\ f(1,p) = 1, & (3.11) \\ f(c_1,p) = 1,\ f(c_3,p) = 1, & (3.12) \\ f(c_2,p) = 1,\ f(c_4,p) = -1, & (3.13) \end{cases} \tag{3.1}$$

(3.11) is the boundary conditions which may eliminate the two parameters k and λ in vector p, (3.12) comes from the two peaks of C and E, (3.13) from the two valleys of D and F, these ensure the every inverse branch has a solution easily. By the six conditions, we get a system of nonlinear equations as (3.2)

$$\begin{cases} F_1(c_1,c_2,c_3,c_4) = 0, \\ F_2(c_1,c_2,c_3,c_4) = 0, \\ F_3(c_1,c_2,c_3,c_4) = 0, \\ F_4(c_1,c_2,c_3,c_4) = 0. \end{cases} \tag{3.2}$$

The expression of (3.2) is so complicated, $F_1(c_1,c_2,c_3,c_4)$ is presented here for demonstration by the symbolic toolbox of matlab 7.0, the rest expressions are ignored .

F1 =(5*c1 + 5*c2 + 5*c3 + 5*c4 - 20*c1*c2 - 20*c1*c3 - 20*c1*c4 - 20*c2*c3 - 20*c2*c4 - 20*c3*c4 + 15*c1^4*c2 + 15*c1^4*c3 + 15*c1^4*c4 - c1^5 + 10*c1*c2*c3 + 10*c1*c2*c4 + 10*c1*c3*c4 + 10*c2*c3*c4 + 10*c1^3*c2*c3 +

10*c1^3*c2*c4 + 10*c1^3*c3*c4 + 10*c1^2*c2*c3*c4 - 20*c1*c2*c3*c4 - 4)/(4*(5*c1*c2 + 5*c1*c3 + 5*c1*c4 + 5*c2*c3 + 5*c2*c4 + 5*c3*c4 + 5*c1*c2*c3*c4 + 1))

It is worthy noting that the solution of (3.2) can not be obtained by the vpasolve and solve function in the symbol toolbox, it could be get only by a numeric solution. The initial point is computed as $P_0 = (-0.72591, -11829, 0.16225, 0.72324)$, it is called the golden initial point because that all the admissible QSSKSs in Table 1 are tested and 86% passed the symbolic convergence conditions and numeric convergence conditions.

3.2 The Application of the NTL

Some of admissible QSSKSs in Table 1 can not get the fixed points of corresponding system of nonlinear equations, however, the genuine solution must exist anyway from the admissibilities. By further research, the error accumulations are found to arouse the numeric solution oscillating. So, we use the high precision algorithm inside the NTL [7], it provides a C++ interface to accomplish arbitrary precision algorithm for real numbers. Here, we presented partly C++ codes for inverse branch Nmap f_N^{-1} with high precision from the NTL, all the double datatype is replaced with class RR. The precision is set to 32 decimal digits for fast speed.

```
   #include <NTL\RR.H>
   RR. SetPrecision(32)
   RR. SetOutputPrecision(32)
void Five_Symbol_Lift::fN(RR c,RR d,RR e,RR f,RR y)
{       RR Gx1,Gx2,Gx3;    RR x1,x2,x3;  int x1sign,x2sign,x3sign;
        x1=d;x2=e; RR m,n,t,dx,y1,y2;
     m=x1;n=x2; if(m>n){t=m;m=n;n=t;}
     y1=f5(m+s1*h);y2=f5(n-s1*h);
       if((y-y1)*(y-y2)>0.0)
     {if(y1<y2){if(y<y1)xn=m+s1*h;else xn=n-s1*h;}
            else{if(y>y1)xn=m+s1*h;else xn=n-s1*h;}
         return;}
       x1=(m+n)/2.0;dx=1.0;
       while(fabs(dx)>s2*h)
         {t=dx;
         dx=(f5(x1)-y)/f_slop(c,d,e,f,x1);
         while(fabs(dx)>0.9*fabs(t)) dx*=0.5;
         x2=x1;
         x1-=dx;
         }
```

```
    xn=x1;
    return;
}
```

For example, the QSSKS $FLDERC$ is hard to get the fixed point, however by the word-lifting technique with high precision algorithm, the corresponding parameters are calculated as,

c1=-0.7488059635519112411927277859919;

c2=-0.3334728931002707921143235258914;

c3=0.18662178995066135119292984232056;

c4=0.77942504988657774327531232773745.

4. Conclusion

Sometimes, the initial point may be proper, iterations for many times always cause the error accumulations, the machine precision can not meet the requirements, the arbitrary precision library such as NTL should be introduced and applied.

The golden initial point is useful for many admissible QSSKSs, however, there are still a few of QSSKSs is sensitive to the initial point, it needs to further study. We judge that two aspects should pay attention to, one is to improve the precision, another is altering newton relaxation factor smartly during the newton iteration process.

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