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Application of Second 0rder Cone Relaxation in Optimal Power Flow Calculation of Distribution Network

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Abstract: The optimal power flow calculation is the important foundation of power grid planning and optimization operation. In this paper, the optimal power flow calculation model for the distribution network with the minimization of all-day active power loss is established. Then the branch power flow constraint is established in combination with the power flow characteristics of the radiant power distribution network, and the output constraint is established by considering the controllable elements in the distribution network, including distributed power supply and discrete and continuous reactive power compensation devices. The model is non-convex nonlinear model. Then the model is transformed into a second order cone programming model with integer variables by the second order cone relaxation, and the YALMIP modeling toolkit is used to call MOSEK commercial solver to solve the model. Finally, an example of IEEE33 node design is given to verify the superiority of the proposed method.

Keywords: Second order cone programming, optimal power flow, distribution network, active power loss.

1. Introduction

The optimal power flow (OPF) of distribution network refers to the optimal operation of the distribution network by controlling the controllable variables in the distribution network under the condition that certain constraints are met. OPF was first proposed by Carpentier in 1962 and has been extensively studied [1]. OPF is a non-convex programming problem which is difficult to solve because of its constraint conditions. At present, OPF solving methods are mainly divided into classical mathematical programming algorithm and intelligent optimization algorithm [2].

Because of OPF's non-convexity, [3] points out that classical programming algorithms (such as Newton's method and interior point method) cannot guarantee the optimality of the solution. Dc power flow method approximates AC power flow constraint to DC power flow constraint and has been widely used in the calculation of optimal power flow of transmission network. However, this method is not applicable to distribution network with high resistance [4]. Intelligent algorithms such as particle swarm optimization and genetic algorithm have been widely used in optimal power flow calculation [5,6]. The intelligent optimization algorithm is not limited by the non-convexity of the model when calculating the optimal power flow. It seeks the optimal solution in the feasible region by setting a certain population and iteration times. However, its iterative process is stochastic and may fall into the local optimal solution. Moreover, repeated iterations lead to a large amount of time consuming in the calculation process and low solving efficiency.

In recent years, many scholars have been exploring efficient methods to solve OPF. With The continuous in-depth study, The second order conic relaxation (SOCR) technology is gradually applied to solve OPF. An OPF model based on branch flow calculation was established [7,8]. For the non-convexity constraints in OPF, SOCR technology was used to relax them into second order cone constraints. The entire OPF model was transformed into the second order cone programming (SOCP), and the global optimal solution could be obtained by solving the model. SOCR technology is adopted in the calculation of the optimal power flow of active distribution network to deal with non-convexity constraints [9], and SOCP is converted into the optimization model to obtain a good solution effect. Moreover, the error between relaxations is analyzed, and the results show that the relaxation error satisfies the calculation accuracy.

The above studies show that SOCR has a strong advantage in OPF calculation. In this paper, the optimal power flow model is first established with the aim of minimizing the all-day active power loss of distribution network, and then the constraint conditions are established based on the branch power flow model. The output constraints of distributed power supply, discrete and continuous reactive power compensation devices and node voltage range constraints are considered in the constraint conditions. The SOCR transforms the model to obtain the mixed integer second order cone programming (MISOCP), which can be solved by existing mature commercial solvers. Based on MATLAB, the YALMIP modeling toolkit is used to invoke MOSEK solver. Aiming at the IEEE33 node design example, the simulation verifies SOCR's effectiveness in the application of optimal power flow calculation, through the comparison with the particle swarm optimization algorithm, the method adopted in this paper has more advantages.

2. Optimal Power Flow Calculation Model

In this paper, one day is taken as an optimization cycle to establish the optimal power flow objective function with the minimum active power loss of all-day distribution network:

$$\min P_{loss} = \sum_{t=1}^{T} \sum_{ij \in E} I_{ij,t}^{2} r_{ij} \tag{1}$$

Where, P_{loss} represents the sum of the active power losses of all branches in the distribution network throughout the day, ij represents the branches connected by node i and node j, E is the set of branches in the distribution network, T is the total number of branches in the whole day, r_{ij} is the resistance of branch ij, and t is the marker of the time period. Therefore, $I_{ij,t}$ and t represent the current of branch ij in the time period.

2.2 Constraint

With the development of active distribution network, more and more controllable units are connected in the distribution network. The optimal operation of distribution network can be achieved by adjusting these controllable units reasonably. In this paper, distributed power supply, discrete reactive compensation device and continuous reactive compensation device are considered. Next, the optimal power flow constraint conditions will be established from the aspects of power flow of distribution network and controllable unit output.

2.2.1 Branch Current Constraint

This paper takes the radiant distribution network as the research object, and selects the operation state of one branch at time t to establish the branch flow model, which is shown in Fig. 1.

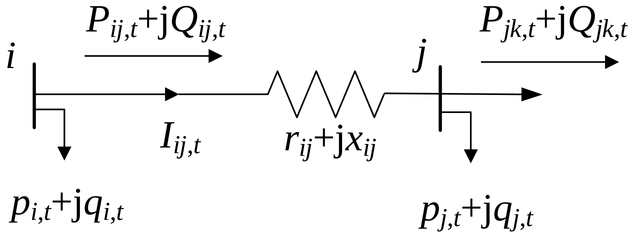

Fig. 1 Branch flow model

This branch shall satisfy the following constraints:

$$U_{j,t}^{2} = U_{i,t}^{2} - 2(r_{ij} P_{ij,t} + x_{ij} Q_{ij,t}) + (r_{ij}^{2} + x_{ij}^{2}) I_{ij,t}^{2} \tag{2}$$

$$p_{j,t} = P_{ij,t} - r_{ij} I_{ij,t}^{2} - \sum_{k:j \to k} P_{jk,t} \tag{3}$$

$$q_{j,t} = Q_{ij,t} - x_{ij} I_{ij,t}^{2} - \sum_{k:j \to k} Q_{jk,t} \tag{4}$$

$$I_{ij,t}^{2}=\frac{P_{ij,t}^{2}+Q_{ij,t}^{2}}{U_{i,t}^{2}} \tag{5}$$

Where, i and j are node Numbers. $U_{i,t}$ and $U_{j,t}$ represent the voltage of node i and j respectively. $p_{i,t}$ and $p_{j,t}$ are the active power injected into node i and j respectively. $q_{i,t}$ and $q_{j,t}$ are reactive power injected into node i and j respectively. $P_{ij,t}$ and $Q_{ij,t}$ represent the first end of the branch ij respectively. $r_{ij}+jx_{ij}$ is the impedance of branch ij. $P_{jk,t}$ and $Q_{jk,t}$ represent the first end of branch jk's active and reactive power respectively, and represent the set of child nodes with node j as the parent node.

2.2.2 Operation Constraints of Distributed Power Supply

If node i is connected to the distributed power supply, then:

$$0\le P_{i,t}^{DG}\le P_{i,t,\max}^{DG} \tag{6}$$

Where, $P_{i,t}^{DG}$ and $P_{i,t,\max}^{DG}$ represent the actual output of the distributed power supply installed at node i in time period t, and the maximum allowable output of the distributed power supply in time period t respectively. This paper assumes that the output of distributed power supply is continuously adjustable.

2.2.3 Constraints of Discrete Reactive Power Compensation Device

If the packet switching capacitor bank (CB) is connected, its reactive compensation amount is a discrete variable. Therefore, CB operation should meet the following constraints:

$$Q_{i,t}^{cb}=n_{i,t}^{cb}Q_{one}^{cb} \tag{1}$$

$$0\le n_{i,t}^{cb}\le n_{i,\max}^{cb} \tag{2}$$

$$n_{i,t}^{cb}\in Z \tag{3}$$

Where, $Q_{i,t}^{cb}$ represents the reactive power compensation capacity of the capacitor at node i in time period t. $n_{i,t}^{cb}$ represents the number of invested CBs, which is taken as an integer. Q_{one}^{cb} is the reactive compensation capacity of each group of capacitors. $n_{i,max}^{cb}$ is the maximum input group number. Z is the set of integers.

2.2.4 Constraints of Continuous Reactive Power Compensation Device

If the static reactive power compensation device (SVC) is connected, its reactive power compensation amount is continuously adjustable. Therefore, SVC operation constraint is as follows:

$$Q_{i,\min}^{svc}\le Q_{i,t}^{svc}\le Q_{i,\max}^{svc} \tag{4}$$

Where, $Q_{i,t}^{svc}$ represents the reactive power compensation amount of SVC device at node i in time period t. $Q_{i,min}^{svc}$ and $Q_{i,max}^{svc}$ respectively represent the minimum value and maximum value of reactive power compensation of SVC device.

2.2.5 Node Voltage Constraint

The load of distribution network has the characteristic of time sequence variation. If distributed power supply and reactive power compensation device with the same characteristic of time sequence are added, the operation state of the system is bound to be more complicated. In order to ensure that the system node voltage can operate within a reasonable range in all time periods, set the node voltage constraint as follows:

$$U_{\min} \le U_{i,t} \le U_{\max} \tag{5}$$

Where, U_{max} and U_{min} represent the upper and lower limits of node voltage operation respectively.

3. Model Transformation Based on Second Order Cone Relaxation

It can be seen that the above constraints include quadratic terms and integer terms, so this optimal power flow problem belongs to mixed integer nonlinear programming problem, and the solving effect of conventional algorithm and intelligent optimization algorithm is not good. Therefore, SOCR is used in this paper to transform the model into a standard second-order cone programming problem that can be solved efficiently. First, variables are introduced:

$$\alpha_{i,t} = U_{i,t}^2 \tag{6}$$

$$\beta_{ij,t} = I_{ij,t}^2 \tag{7}$$

Where, $\alpha_{j,t}$ and $\beta_{j,t}$ represent the square of branch *ij* current of node *i* voltage in time period *t*. Then the optimal power flow objective (1) and constraint conditions (2)-(5) and (11) can be written as:

$$\min P_{loss} = \sum_{t=1}^{T} \sum_{ij \in E} \beta_{ij,t} r_{ij} \tag{8}$$

$$\alpha_{j,t} = \alpha_{i,t} - 2(r_{ij} P_{ij,t} + x_{ij} Q_{ij,t}) + (r_{ij}^2 + x_{ij}^2)\beta_{ij,t} \tag{9}$$

$$p_{j,t} = P_{ij,t} - r_{ij}\beta_{ij,t} - \sum_{k:j \to k} P_{jk,t} \tag{10}$$

$$q_{j,t} = Q_{ij,t} - x_{ij}\beta_{ij,t} - \sum_{k:j \to k} Q_{jk,t} \tag{11}$$

$$\beta_{ij,t} = \frac{P_{ij,t}^2 + Q_{ij,t}^2}{\alpha_{i,t}} \tag{12}$$

$$U_{i,t,\min}^2 \le \beta_{i,t} \le U_{i,t,\max}^2 \tag{13}$$

It can be seen that (14) is a linear function, (15)-(17) and (19) are linear constraints, while (18) is a nonlinear equality constraint. SOCR is used to process (18) at this time [10]:

$$\beta_{ij,t} \ge \frac{P_{ij,t}^2 + Q_{ij,t}^2}{\alpha_{i,t}} \tag{14}$$

After equivalent transformation, (20) can be written into the standard second-order cone form:

$$\left\| \begin{matrix} 2P_{ij,t} \\ 2Q_{ij,t} \\ \beta_{ij,t} - \alpha_{i,t} \end{matrix} \right\|_2 \le \beta_{ij,t} + \alpha_{i,t} \tag{15}$$

Where, ||||2 represents the two-norm. Through the above transformation, the optimal power flow model in this paper is transformed into a second-order cone programming, and its complete expression is:

$$\begin{gathered} \min P_{loss} = \sum_{t=1}^{T} \sum_{ij \in E} \beta_{ij,t} r_{ij} \\ s.t. \quad (15)-(17),(19),(21),(6)-(10) \end{gathered} \tag{16}$$

Because CB running constraints contain integer variables, the model is MISOCP. It can be seen that SOCR processing actually relaxes the scope of constraints for the efficient solution of the model, which will cause certain errors to the model. Here, the relaxation error is defined as:

$$err_{ij,t} = \beta_{ij,t}\alpha_{i,t} - P_{ij,t}^2 + Q_{ij,t}^2 \tag{17}$$

The validity of SOCR transformations has been fully studied [11], and the results show that the model is still accurate after SOCR transformations under fairly loose conditions. In this paper, this error is analyzed in an example to prove the validity of SOCR for model processing.

4. Model Solving Process

For MISOCP, the existing mature commercial solvers can be used to solve the problem quickly and accurately. In this paper, based on the MATLAB platform, the YALMIP modeling toolkit is used to model the model and MOSEK solver is called to solve it. The program implementation process is as follows:

i. Set basic parameters. Including distribution network topology, branch impedance, distributed power supply, CB, SVC and other parameters.

ii. Define the timing control variables of the optimal power flow model. Including distributed power supply, CB, SVC output related variables, branch current squared term, node voltage squared term, line transmission active and reactive power, etc.

iii. Set constraints for each period.

iv. Call MOSEK solver to solve and output the result.

5. Data Collection by Questionnaire Survey

5.1 Parameter Setting

In this paper, a simulation analysis is carried out for IEEE33 node design example. IEEE33 node is shown in Fig. 2, and relevant data references are given [12].

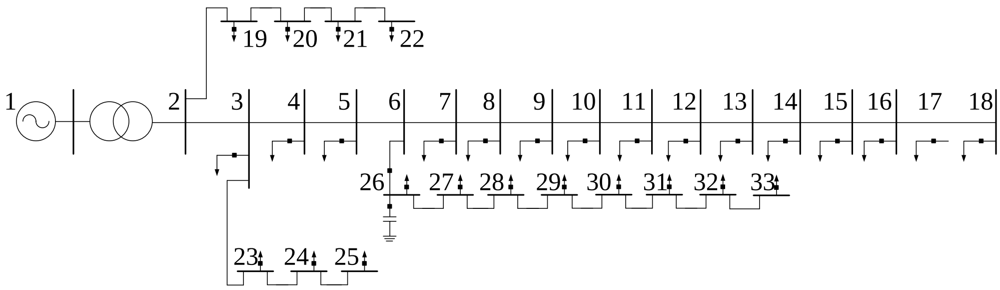

Fig. 2 IEEE33

Two distributed power sources, photovoltaic (PV) and wind turbine (WT), are considered. PV was installed at node 8 with an installed capacity of 1.5MW.WT is installed at node 12 with an installed capacity of 1MW.Node 18 is equipped with CB, which has a capacity of 50kVAR for each group, a total of 10 groups. SVC is installed on node 31, and the compensation range is -0.2mvar-1 Mvar. The operating range of node voltage is 0.93-1.07 PU. The model in this paper optimizes the operation condition of distribution network 24 hours a day. The timing prediction of load, PV and WT is shown in Fig. 3. The load in each period is the IEEE33 node base load multiplied by the corresponding time sequence value. The maximum PV and WT output in each period is the respective installed capacity multiplied by the corresponding time series.

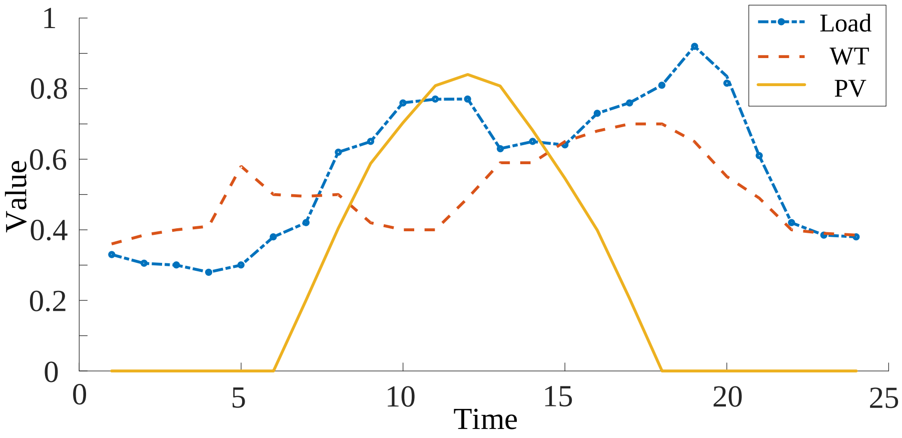

Fig. 3 Timing prediction about load, WT and PV

5.2 Results analysis

By calling MOSEK solver to solve the model, the sum of the work loss in each period is 537.57kW, the active power loss before system optimization is 1691.20kW, and the active power loss after optimization is reduced to 31.79% before optimization. The comparison of active power loss before and after optimization of each period of the system is shown in Fig. 4. It can be seen that the active power loss curve of each period of the system is similar to the load timing value, indicating that the active power loss increases with the increase of the load and decreases with the decrease. The method adopted in this paper can coordinate and control the output of distributed power supply and reactive power compensation device, greatly reduce the active power loss of the system, and improve the economy of distribution network operation.

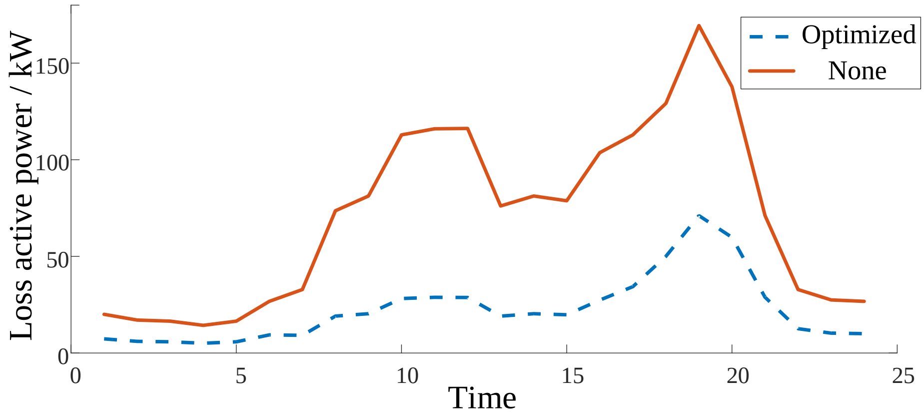

Fig. 4 Comparison of active power loss in each period

In this paper, the total output (PV and WT) and reactive power compensation in each period obtained by solution are shown in Fig. 5. Each unit can coordinate and optimize the operation within the output constraints. In order to optimize the active power loss of the system, when the maximum output of distributed power supply accounts for a large proportion of the load, the output is reduced. CB and SVC also adjust the reactive power compensation according to the load changes.

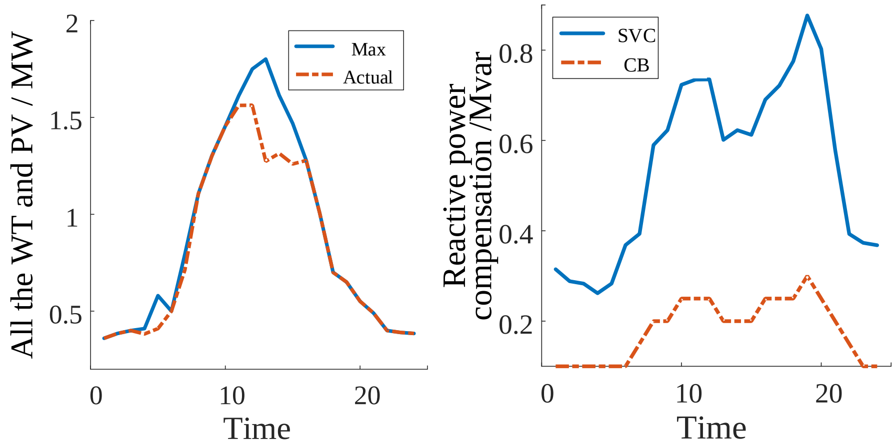

Fig. 5 Output of each period

In order to analyze the accuracy of SOCR's treatment of the model, the calculation method of (23) is taken as a reference, and the relaxation error of each branch at each period is obtained, as shown in Fig. 6. It can be seen that the maximum magnitude of relaxation error is 10-6, which meets the operation requirements. The SOCR relaxes the model effectively.

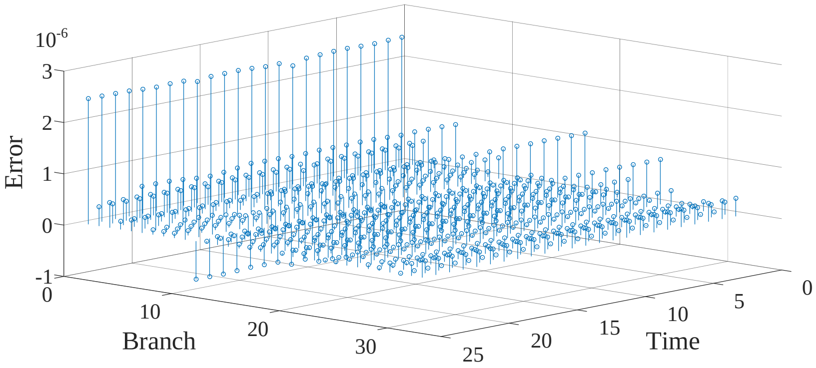

Fig. 6 Relaxation error of each branch in each period

In order to further verify the effectiveness of the solution method in this paper, optimization analysis is carried out in the 12th period. Using the method in this paper, the solution time is 0.372 seconds, and the active power loss in this period is calculated as 28.74kW. At the same time, particle swarm optimization (PSO) is used to calculate the model and set different parameters of PSO. Compare the calculated results, as shown in table 1.

Table 1 PSO results under different parameters

Case	Particle population number	Iterations number	Computing time/S	Active power loss/kW
1	15	15	0.642	28.82
2	20	20	1.099	28.78
3	30	30	2.521	28.75

It can be seen from Table 1 that, due to the randomness of the iterative process of PSO, its solution result is unstable and depends on the population number and iteration times. With the increase of population number and iteration times, the solution result of PSO is better, but the solution time is longer. By comparing with PSO, it can be found that the method presented in this paper has the advantages of fast and accurate model solving.

6. Conclusion

In this paper, the optimal power flow calculation model based on branch power flow constraint is established with the aim of minimizing the all-day active power loss of distribution network. The model also takes into account the operation constraints of distributed power supply and discrete and continuous reactive compensation devices. This model is a nonlinear and non-convex programming model with integer variables. The model is transformed using SOCR to make it a MISOCP program that can be efficiently solved. Finally, the YALMIP modeling toolkit is used to invoke MOSEK commercial solver to solve the model efficiently. Simulation results show that:

i. Coordinating and controlling the controllable units in the distribution network can greatly reduce the active power loss of the distribution network and improve the economy of the system operation.

ii. An example is given to analyze the relaxation error, and the results show that the relaxation error can meet the operation requirements, and SOCR is effective in the processing of the model;

iii. By comparing with the solution result of PSO, it can be found that the method adopted in this paper can obtain the optimal solution stably.

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