



## **Power Control of Wireless Network Based on Graph Convolution Network**

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**Abstract:** In this paper, Inspired by the recent successes of deep learning in many application domains. we study deep-learning-based power control methods for Multi-user interference network, in order to ensure the quality of service (QoS) requirement of each user and the maximum transmission power constraint of each node. which to boost the throughput of the whole network. Due to the inter-user interference, to solve the problem of maximum sum rate in multi-user interference channel, the considered problem is nonconvex and NP-hard. Different from traditional optimization techniques, we rely on the deep-learning (DL) method to find the solution adaptively, we proposed a power control framework based on graph convolution neural network (GCN) to learn the optimal power control in an unsupervised manner. The proposed framework firstly transforms the wireless interference channel into graph data structure, and proves the disorder of the interference channel. Then, according to the characteristics of power allocation criteria, the GCN network structure is constructed. Due to the QoS constraints in the optimization problem makes things even more complicated. To tackle this difficulties, we adopt to add a penalty term in the loss function to solve the rate constraint problem of users. Simulations demonstrate the effectiveness of the proposed GCN-based power control method.

**Keywords:** Deep learning, quality of service, sum rate, graph convolution neural network, power control.

### **1. Introduction**

In recent years, the number of wireless mobile users has increased due to the rapid development of mobile Internet and the continuous updating of intelligent terminal technology. This tendency is expected to continue over the next few years[1]. Next-generation mobile networks are committed to developing key technologies to meet the needs of high data rates, low power consumption, and massive connectivity,

ensuring the quality of service across the network (QoS)[2]. With the increasing popularity of wireless equipment, more and more users appear in wireless communication, which makes the interference between users become more and more serious, increasing the demand for transmit power control of interference network. The solution to this problem has a significant impact on today's wireless network.

Effective resource allocation is becoming increasingly important in wireless network optimization. However, many of these common resource allocation problems are nonconvex, such as power control [3][4][5], which is extremely difficult to calculate. Most previous research on resource management has treated resource allocation as an optimization problem, with researchers employing corresponding optimization tools to solve these problems. The problem of power control becomes more complicated as more QoS constraints are added. For example, paper[6] proposed the Geometric Programming (GP) method to solve the problem of maximizing the weighted sum rate while adhering to explicit QoS constraints. The SCALE algorithm was proposed in [7] to solve a series of approximated convex problems. In [8], A new method based on sequential quadratic programming (SQP) is proposed to solve the convex optimization problem, which can improve the sum rate of a small cell network while ensuring the QoS requirements of each user and the maximum transmit power constraint of each node. These algorithms need a lot of iterative calculations to converge, and complex matrix operations, such as singular value decomposition, matrix inversion and so on. Usually included in each iteration. Therefore, applying them to real-time systems may encounter many great challenges due to high computational complexity.

Deep learning has had great success in computer vision, natural language processing and some other applications. Recent results also show that deep learning can be a promising tool in solving difficult communication problems. The author in [9], to solve the power control problem, proposed a multi-layer perceptron (MLP) to approximate the input-output mapping of the classic weighted minimum mean square error (WMMSE) algorithm [10] to speed up the computation. However, because the primary goal in this case was to regenerate a WMMSE-based scheme, the achievable capacity of the DNN-based scheme cannot be greater than that of the WMMSE-based scheme. To solve the above problems, the paper [11] proposed a DL-based method that uses unsupervised learning and ensemble learning to find the global optimal solution, which users with poor channel conditions may be turned off. The author in [12] proposes the use of graph neural network architectures to analyze power control and beamforming problems. Under the co-channel deployment, some of the users may experience significantly greater performance degradation than others. Thus, a guaranteed quality of service (QoS) becomes necessary[6]. The minimum QoS refers to each user's minimum rate requirement. The optimization maximizes the overall rate of the system

under the guaranteed minimum rate for each user.

In this paper, we study power control framework based on GCN for Multi-user interference network. Specifically, we formulate a sum rate maximization power control problem subject to QoS constraints for the transmitter and the receiver. The considered sum rate maximization problem is nonconvex and difficult to solve. In order to develop a GCN-based based solving method, we propose creating a new loss function that penalizes the violation of the constraint to solve the minimum rate constraint, which penalty term is used to encourage network output to meet the rate constraint. Then, GCN constructed leveraging the unsupervised learning strategy. The GCN adjusts its parameters by minimizing the loss function computed from the negative value of the objective function. Using this unsupervised learning strategy, the constructed GCN network output optimized power when channel state information is input. The effectiveness of this method is confirmed by simulation results.

The rest of this paper is organized as follows. In Section II, we present the system model. Then, a GCN-based power control framework is devised in Section III. Simulation results are presented in Section IV. Finally, we draw conclusions in Section V.

## 2. SYSTEM MODEL

We consider a general K-user single-antenna interference channel as shown in Fig. 1. It is assumed that all transmitter-receiver pairs are synchronized and share the same narrowband spectrum. The received signal at the k-th receiver is given by

$$y_k = h_{kk}x_k + \sum_{j \neq k} h_{kj}x_j + n_k \quad (1)$$

Where  $K = \{1, 2, \dots, k\}$  denotes the set of transmitter-receiver pairs.  $h_{kk} \in \mathbb{C}$  denotes the direct-link channel between the k-th transmitter and receiver,  $h_{kj} \in \mathbb{C}$  denotes the cross-link channel between transmitter k and receiver j,  $x_k \in \mathbb{C}$  denotes the signal transmitted by the k-th transmitter, and  $n_k \sim \mathcal{CN}(0, \sigma_k^2)$  denotes the receiver noise, which is independent across both time and users.

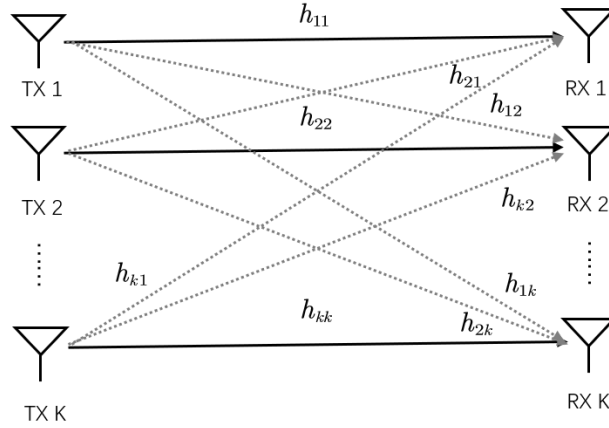


Fig. 1 The K-user interference channel.

According to the settings on, The k-th receiver's signal-to-interference-plus-noise ratio (SINR) is given by

$$SINR_k = \frac{|h_{kk}|^2 p_k}{\sum_{j \neq k} |h_{kj}|^2 p_j + \sigma_k^2} \quad (2)$$

Where  $p_k$  is the power of the k-th transmitter,  $\sigma_k^2$  denotes the additional noise power of the k-th receiver.

The achievable rate of the k-th receiver is given by

$$R_k(\mathbf{P}) = \log_2(1 + SINR_k) \quad (3)$$

Where  $\mathbf{P} = (P_1, P_2, \dots, P_K)^T$  denotes the joint the power allocation vector of all users.

The maximization of the sum rate problem is formulated as

$$\begin{aligned} & \underset{\mathbf{p}}{\text{maximize}} \quad \sum_{k=1}^K R_k(\mathbf{P}) \\ & \text{subject to :} \\ & C1: R_k(\mathbf{P}) \geq r_{k,\min}, \forall k \in K \end{aligned} \quad (4)$$

$$C2: 0 \leq P_k \leq P_{\max}, \quad \forall k \in K$$

Where  $r_{k,\min}$  is the minimum required rate of the k-th receiver. we define  $r_{\min} = (r_{1,\min}, r_{2,\min}, \dots, r_{K,\min})$  and  $P_{\max} = 1$ , The problem (4) is nonconvex because both the objective and QoS constraints of all users are nonconvex, making obtaining the global optimal solution NP-hard[13][14].

If the target rate is large, the problem may be unsolvable. It is not difficult to develop a standard for check its feasibility[4][14].For a receiver k, its received SINR value needs to be at least  $\gamma_{k,\min} = 2^{r_{k,\min}} - 1$ , to satisfy its minimum rate requirement. Defined

matrix  $\mathbf{A}$  is given by

$$A_{k,j} = \begin{cases} 0, & k = j \\ \frac{\gamma_{k,\min} \|h_{kj}\|^2}{\|h_{kk}\|} & k \neq j \end{cases} \quad (5)$$

Where  $A_{k,j}$  denotes the (k,j)-th element of  $\mathbf{A}$ , If the maximum eigenvalue of  $\mathbf{B}$  is less than 1, there is possible to find a feasible solution as

$$\tilde{\mathbf{P}} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{u} \quad (6)$$

where  $\mathbf{I}$  denotes an  $K \times K$  identity matrix and  $\mathbf{u}$  denotes a  $K \times 1$  column vector with the  $i$ -th element  $u_i$  as

$$u_i = \frac{\gamma_{k,\min} \sigma_k^2}{\|h_{kk}\|} \quad (7)$$

In the power allocation  $\tilde{\mathbf{P}}$ , if all elements are in range between 0 and  $P_{\max}$ . As mentioned above, the power allocation  $\tilde{\mathbf{P}}$  is a feasible solution of the maximization of the sum rate problem.

Different from the traditional algorithm using convex optimization method, we propose a GCN-based method in the next section to solve the problem of maximizing the sum rate.

### 3. Power Control Scheme Based on Graph Convolution Network

In this section, we first create a basic structure of GCN network, followed by an introduction to the feasibility of GCN in power allocation. Then we model a  $K$ -user interference channel and analyse the permutation invariance of interference channels in power allocation problem. Under the graph model representation of interference channel. We propose a node update mechanism and construct the corresponding GCN network structure to solve the optimization problem of the above power criterion.

#### 3.1 The Fundamental Structure of a Graph Convolution Network

GCN, like MLP or CNN, has a layer structure. In GCN, the update rule at vertex  $v$  of layer  $l$  is expressed as follows:

$$g_v^{(l)} = \text{AGGREGATE}^{(l)}(\{h_u^{(l-1)} : u \in N(v)\}, \{\eta_x : x \in \varepsilon(v)\}) \quad (8)$$

$$h_v^{(l)} = \text{COMBINE}^{(l)}(h_v^{(l-1)}, g_v^{(l)}) \quad (9)$$

Where  $N(v)$  denotes the set of neighbor nodes of node  $v$ ,  $\varepsilon(v)$  denotes the set of all edges with  $v$  as an end point,  $\text{AGGREGATE}(\cdot)$  and  $\text{COMBINE}(\cdot)$  are two functions,

$\eta_x$  denotes the characteristics of edge  $x$ ,  $h_v^{(l)}$  denotes the  $l$ -th layer's out feature of vertex  $v$ ,  $g_v^{(l)}$  is an intermediate variable.

### 3.2 Power Allocation Framework Based on GCN

When we apply GCN to wireless resource allocation, an important factor is to consider the geometric properties of interference channels. According to the mapping relationship between channel matrix and optimal power allocation, the geometric properties of interference channels can be verified.

For a given  $k$ , let  $f_k(\cdot)$  denotes the mapping relationship between the channel matrix and transmission power of the  $k$ -th transmitter, i.e.,  $p_k^* = f_k(\mathbf{H})$ , then  $\mathbf{\Pi} \in \{0, 1\}^{K \times K}$  denotes any permutation matrix satisfying  $(\mathbf{\Pi}^T \mathbf{H} \mathbf{\Pi})_{kk} = h_{kk}$ , then an equation is given by

$$p_k^* = f_k(\mathbf{H}) = f_k(\mathbf{\Pi}^T \mathbf{H} \mathbf{\Pi}) \quad (10)$$

The Equation (10) shows that the interference channel has disorder or permutation invariance, which means that what matters is the set of interference channel coefficients rather than their order. The permutation invariance of the channel matrix in power allocation demonstrates that considering neighboring components is pointless because there is no correlation between elements after arrangement. The interference channel is typically expressed as a channel matrix of Euclidean data structure in power allocation based on MLP and CNN. The introduction of the graph data structure demonstrates that node  $V$  also only pays attention to the set  $N(v)$  of surrounding nodes and ignores the order, indicating that the interference channel is acceptable for representation by a non-Euclidean graph data structure.

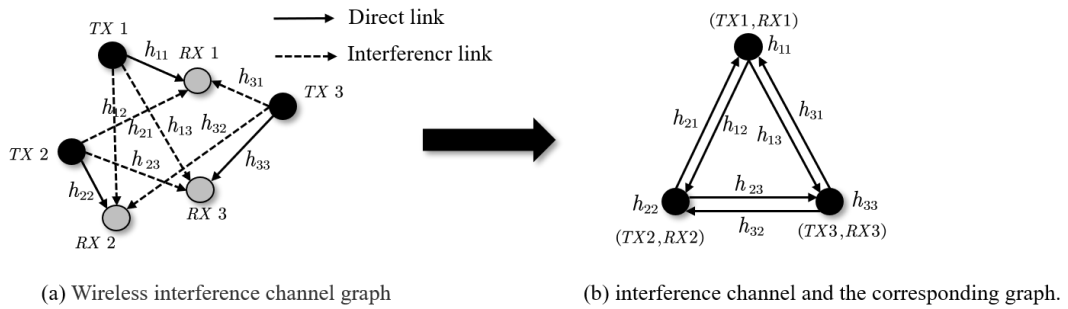


Fig. 2 The data structure of interference graph

In Fig.2(a), the pairs between the transmitter and receiver are regarded as a node, each link as an edge, and the weight of each edge is the channel gain. Consider each transmission pair to be a node and each interference link to be an edge, a graph model for wireless interference channel is constructed as shown in Fig2(b). The channel gain

of the corresponding communication link and the channel gain between two nodes of the interference link, respectively, determine the weights of features and edges. As a result, any two nodes have two directional edges. In this way, the original interference channel model is depicted in fig. 2(b) as a weighted directed graph.

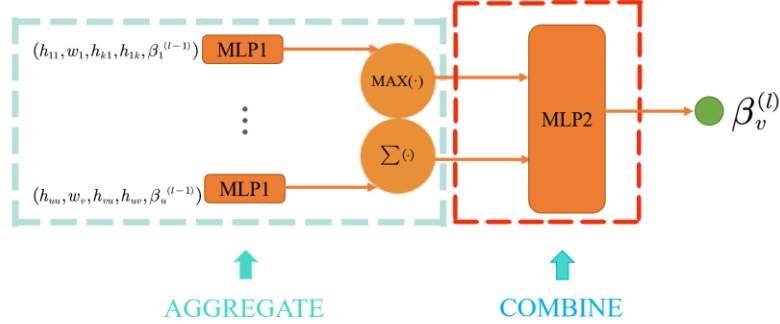


Fig. 3 The structure of power allocation in the proposed GCN

Based on the graph model of wireless interference channel mentioned above, we adopt GCN to power allocation. For the node  $v$ , the Fig. 3 depicts the power allocation structure and the update rule is

$$\eta_{u,v}^{(l)} = MLP1(h_{u,v}, h_{v,u}, h_{u,u}, \beta_u^{l-1}) \quad (11)$$

$$\alpha_v^{(l)} = CONCAT(MAX_u\{\eta_{u,v}\}, \sum_u \eta_{u,v}), u \in N(v) \quad (12)$$

$$\beta_v^{(l)} = MLP2(\alpha_v^{(l)}, h_{v,v}, \beta_v^{(l-1)}) \quad (13)$$

Where MLP1 and MLP2 denote two different MLPs, CONCAT denotes the operation that concatenates two vectors together,  $\eta_{u,v}^{(l)}$  denotes the feature vector of the edge connecting vertex  $u$  and vertex  $v$ ,  $\beta_v^{(l)}$  denotes the output characteristics of  $l$ -th layer of GCN at vertex  $v$ . In the problem of power allocation,  $\beta_v$  denotes the output power of the node  $v$ . For the initial value of  $\beta_v^{(0)}$  as initialize to  $P_{\max}$ . The network structure of MLP1 and MLP2 are shown in Tables 1 and 2, respectively.

Table 1 MLP1 network structure of AGGREGATE function

Neural network layer	Content	output dimension
Input	Node characteristics and edge characteristics of neighbors	32+2=34
Hidden layer 1	Linear(34,16)+ReLU	16
Hidden layer 2	Linear(16,1)	1
Hidden layer 3	Sigmoid	1

Table 2 MLP1 network structure of COMBINE function

Neural network layer	Content	output dimension
Input	Node characteristics and edge characteristics of neighbors	10
Hidden layer 1	Linear(4,32)+ReLU	32
Hidden layer 2	Linear(16,32)	32
Hidden layer 3	Linear(16,32)	32

The final output of GCN is the transmission power of the transmission pair that represented by the node, thus the output dimension is set to 1. Meanwhile, in order to limit the output power to the maximum output power, the output layer activation function of MLP 2 adopts Sigmoid activation function. While the activation functions of other layers of MLP1 and MLP2 are linear rectifier units (ReLU).

We adopt the adopted training method is unsupervised manner, however, the loss function is the key of the unsupervised learning process. As a result, only the channel matrix sample is required during the training process, and no label is required. We define the loss function according to the objective function of (4), which is written as

$$loss = E \left[ - \sum_{k=1}^K R_k(h, \theta) + \lambda \cdot \sum_{k=1}^K \text{ReLU}(r_{k,\min} - R_k(h, \theta)) \right] \quad (14)$$

Where  $\theta$  denotes the parameters of the neural network. The loss function that can be directly optimized by stochastic gradient descent. The penalty term is used to encourage network output to meet the minimum rate constraint. If  $r_{k,\min} > R_k(h, \theta)$ , i.e. the rate constraint is not met,  $\text{ReLU}(r_{k,\min} - R_k(h, \theta)) > 0$  and the corresponding penalty term will force the network parameters to be updated in the direction that satisfies the constraint. On the contrary, if  $r_{k,\min} \leq R_k(h, \theta)$ ,  $\text{ReLU}(r_{k,\min} - R_k(h, \theta)) = 0$  and the network training will be unaffected by the corresponding penalty item. In this case, the training process will focus on improving the system sum rate by making the network output meet the rate constraints of other receivers. The scaling factor  $\lambda$ , which is a hyperparameter, balances the trade-off between different terms in the loss function. If it is too large, the network will focus on meeting the rate limit at the expense of sum rate performance; If it is too small, the network may be unable to generate a feasible power profile.

Different from the MLP-based method, the GCN-based method needs to transform the channel matrix into the graph data structure. The establishment, training and testing of GCN model are based on Pytorch Geometrics [15].

#### 4. Simulation Results

In this section, we present the simulation results to demonstrate the effectiveness of



the proposed power control scheme. In the experiments, the channel state information service is distributed in this experiment using a complex Gaussian with a zero mean value and a single bit variance. i.e.,  $h_{kj} \sim \mathcal{CN}(0, 1), \forall k, j = 1, 2, \dots, K$ . Without loss of generality, the noise power is normalized, i.e.,  $\sigma^2 = 1$ .

We evaluated the proposed power control scheme, considering the cases of  $K=3$  and  $K=6$ . In order to compare the performance, we compare it to the Geometric Programming (GP) method and Exhaustive search method proposed in reference [6]. The GP-based method, as we all know, is a high-performance algorithm for solving explicit QoS constraints. It approximates the original non-convex problem as a series of convex problems and solves these convex problems iteratively using geometric programming until the solution converges.

We generate 10000 channel samples to train the GCN network. Due to the high complexity of GP, the results in this section are obtained by simulating 1000 channel samples. Thus the test dataset contains 1000 network realizations.

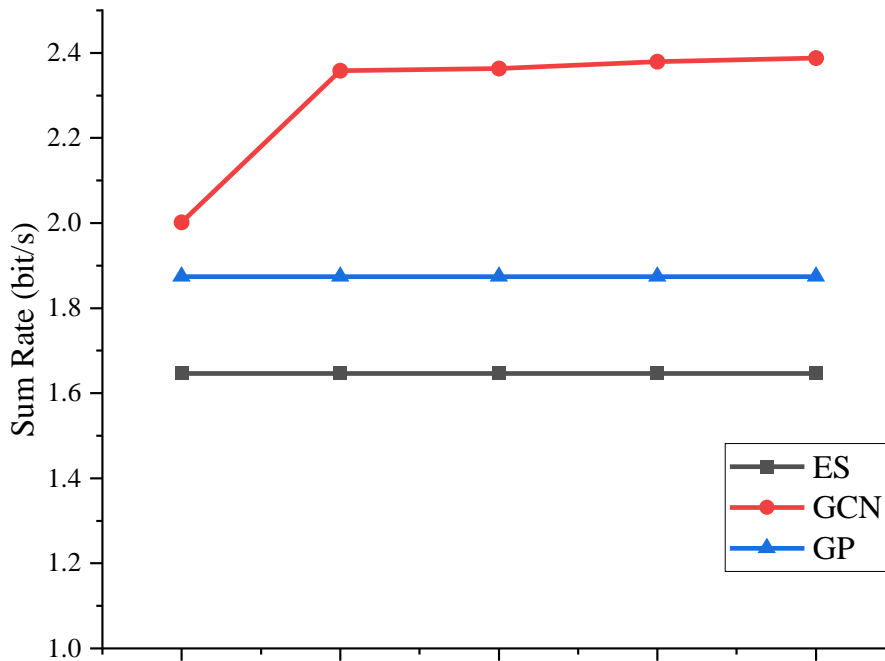


Fig. 4 Average Sum rate comparison among 3-users from different methods

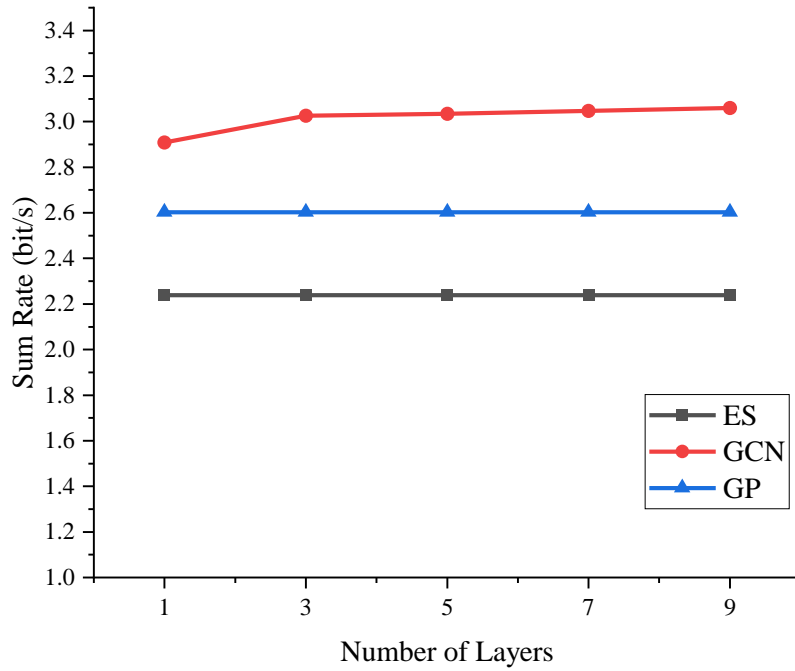


Fig. 5 Average Sum rate comparison among 6-users from different methods

#### 4.1 Performance Comparison

In Fig.4, we compare the sum rates of  $K=3$  under different minimum rate constraints and study the impact of the number layers. we set the numbers of layers as  $\{1,3,5,7,9\}$ . We then study the impact of the number of layers and observe the test performance of the proposed scheme. The loss function is constructed by (14) and the scaling factor  $\lambda$  of the loss function is set to 10. Each user is set a minimum rate to ensuring a minimum requirement of QoS, We set these value as  $r_{\min} = (0.5, 0.3, 0.2)$ , the performance of the GCN is shown in Fig.4. We can see 1-layer GCN outperforms GP by 13% and the performance of 3-layer GCN is approximately 26% higher than that of GP. Similarly, the performance is higher than the exhaustive search method. Intuitively, the GCN with a larger number of layers will perform better, because  $m$ -hop information is gathered if a  $m$ -layer GCN is used. This is not surprising given that GCN captures more information with more layers. We also see a significant performance improvement from 1-layer GCN to 3-layer GCN, which demonstrates that multi-hop information is critical for the performance.

#### 4.2 Expandability and QoS Assurance Capability

Under different minimum rate constraints, we demonstrate the achieved sum rate of the proposed methods in a system with  $K = 6$ . In the simulations. We set the QoS threshold of each user as  $r_{\min} = (0.5, 0.1, 0.1, 0.1, 0.1, 0.1)$  and  $P_{\max} = 1$ . In Fig.5, It is shown that the proposed GCN also achieves a better performance than GP. As a

result, leveraging the graph structure of the interference channel appears to be beneficial for maintaining good performance when the network size is large.

Table 3 The transmission power and achievable rate of each user with K=3

QoS rate threshold per user	(0.5 , 0.3 , 0.2)
Output power rate vector	(0.5127,0.9962,0.3752)
Rate of each user	(0.6889 , 0.3519 , 0.2165)

Table 4 The transmission power and achievable rate of each user with K=6

QoS rate threshold per user	(0.5,0.1,0.1,0.1,0.1,0.1)
Output power rate vector	(0.8725,0.6299,0.5921,0.5187,0.6759,0.5239)
Rate of each user	(0.5505,0.1196,0.1107,0.1207,0.1194,0.1061)

In the above Table 3 and Table 4, in the case of K=3 and K=6, according to the set user QoS rate constraint and the transmission power constraint, the optimized transmission power vector output by the network satisfies the transmission power constraint, and the rates of all users are higher than the corresponding predefined QoS values. It is concluded from this example that the proposed method is scalable and can be extended to more users' systems.

## 5. Conclusion

In this paper, We studied the sum rate maximization power control problem to the QoS constraints of the users for the K-user interference channel. We propose to transform the wireless interference channel into a corresponding graph data structure for power control using unsupervised learning graph convolutional neural network. Under the QoS rate constraints of users, the penalty term is added to the loss function to solve the rate constraint problem. Simulations demonstrate the effectiveness of the proposed power control method in guaranteeing the QoS of the users when attempting to maximize the sum rate of the whole network.

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